The Forms of Gold. Work II: Idealized Gold

Gold in an Ideal World and the Refrigerator Problem

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Abstract

This paper is Work II of *The Forms of Gold*, a theoretical investigation of commodity money and capitalist reproduction. It develops the concept of Idealized Gold: a freely reproducible money commodity possessing Commodity Value, Monetary Character, Unlimited Reproducibility, and Monetary Persistence. Kautsky's account of capital and labour moving into Gold production provides an initial point of departure. The paper asks what becomes of this endogenous regulating mechanism once the independent natural constraints of Real Gold production are removed. It distinguishes Direct Monetary Realization from the Current Reproduction Value of Gold and shows why, under unchanged value conditions, Gold Capital has no unilateral incentive to raise physical Gold productivity. General Technical Progress nevertheless cheapens Gold Constant Capital and raises the Gold Rate of Valorization. Before the physical minimum of Gold production is reached, this cheapening induces Compensating Technical Deterioration: Gold productivity falls sufficiently to preserve the Current Reproduction Value while the Rate of Valorization rises. Once the Minimum Viable Gold Technique becomes binding, further cheapening produces Technical Devaluation of Gold itself. From this development follow the Profit-Rate Crossing, Goldization, Labour Competition, the moving Labour-Compensation Threshold, and the Reproductive Boundary. If Idealized Gold remains operative, the system reaches either the Terminal Economic Boundary of positive capitalist valorization or Reproductive Breakdown. If it is abandoned economically or institutionally before either event, the Idealized-Gold model ends with the disappearance of its money commodity. The Refrigerator Theorem and its termination corollary formalize this result: Capitalism with Idealized Gold cannot persist as a stable terminal configuration. Historical Gold Rushes are considered as cases in which Real Gold temporarily approaches the conditions of Idealized Gold. Bitcoin is interpreted as Digital Real Gold whose reproduction remains independently constrained by protocol. The comparison with unrestricted digital Fan Tokens then clarifies the distinction between Commodity Value and reproductive constraint and provides the transition to the analysis of Fiat and Credit Money developed in Work III, *Ideal Gold*.

Keywords: Idealized Gold; commodity money; Monetary Persistence; Current Reproduction Value; Monetary Privilege; direct monetary realization; Compensating Technical Deterioration; Minimum Viable Gold Technique; technical devaluation; Goldization; Refrigerator Problem; capitalist reproduction; valorization; material reproduction; labour competition; Marxian political economy; Polar Marxism; Real Gold; Material Real Gold; Digital Real Gold; Gold production; Gold Rushes; Kautsky; Bitcoin; Proof of Work; reproductive constraint; Fan Tokens; Fiat Money; Credit Money
JEL Codes: B51, E40, E42, E25, O33, J31

0. Methodological Foundations of Economic Dynamics

The analysis developed in this work begins from a distinction between **economic reality** and its **mathematical representation**. Economic processes are realized through concrete changes in production, investment, replacement, employment, labour cost, technical implementation, and the allocation of capital. Mathematical representation may simplify, compare, and clarify these processes, but it must not substitute its own formal properties for the economic mechanisms being represented.

The following methodological structure governs all dynamic arguments developed in this work.

Fundamental Methodological Principle 1 (Economic Primacy over Mathematical Representation)**.** The economic mechanism is logically prior to its mathematical representation. Mathematical notation may represent, simplify, or approximate an economic process, but it may not introduce an economic mechanism, restriction, terminal condition, or possibility that has not been economically specified.

A continuous function, derivative, or limiting expression may be useful for representing direction, comparative movement, functional dependence, or a general tendency. But properties arising solely from the chosen mathematical representation are not automatically properties of the economic process itself.

In particular, a mathematical possibility does not become an economic possibility merely because a function can be constructed that exhibits it. If an economic process accelerates, decelerates, stops, reverses, or becomes negligible, the model must contain an economic reason for that development.

Economic Postulate 1 (Finite Discrete Economic Change)**.** Realized economic change occurs through finite changes between successive economic states. An individual change may be very small, but it remains a finite realized change. The model does not interpret economic adjustment as being physically composed of infinitesimal steps, nor does it assume that successive realized changes automatically become indefinitely smaller.

For an economic magnitude z, the realized movement between two successive states may therefore be represented directly as:

$$\Delta z_n = z_{n+1} - z_n.$$

The magnitude of Δz_n may vary from one state to another. The postulate does not impose a universal minimum step and does not require all economic processes to move by equal increments. It states only that actual economic adjustment consists of realized finite changes. A progressive contraction of those changes must itself have an economic cause; it is not implied merely by the availability of an asymptotic mathematical representation.

Economic Postulate 2 (Objective Economic Scale)**.** Every economic process possesses a scale determined by the material, technical, institutional, and social conditions through which that process is actually realized. The magnitude and speed assigned to an economic change must therefore remain commensurate with the economic mechanism being represented.

The Objective Economic Scale is not a universal numerical constant. Different economic processes operate at different magnitudes and over different intervals. Capital reallocation, replacement of productive equipment, technical diffusion, changes in labour cost, and ordinary commodity production need not proceed at the same speed or by the same increments.

The postulate instead excludes purely formal distortions of an economic mechanism. An economically active process cannot be assigned an arbitrarily vanishing rate merely because such a rate is mathematically admissible, unless the model identifies an economic mechanism producing that contraction. Likewise, a process cannot be assigned an arbitrarily explosive rate without an economic mechanism capable of producing it.

The scale of economic change is therefore determined by the economic mechanism rather than by the unrestricted freedom of the mathematical function used to represent it.

Economic Postulate 3 (Economic Finitude)**.** Every realized economic magnitude is finite at every realized economic state. Mathematical infinity is therefore not itself an attainable economic magnitude.

An expression such as $z_n \to \infty$ does not mean that an actually existing economic magnitude becomes literally infinite. It means that, within the mechanism and range represented by the model, no internal finite upper bound has yet been generated by that mechanism.

The economic interpretation of an unbounded mathematical tendency is therefore:

$$\text{absence of an internal modeled bound} \neq \text{attainment of economic infinity.}$$

In an actual economy, continued movement must remain subject to finite material, technical, social, institutional, or other economically relevant conditions. If such a condition becomes relevant to the analysis, it must be introduced as an economic mechanism rather than inferred from the concept of mathematical infinity itself.

Interpretive Principle 1 (Economic Zero)**.** An economic magnitude may become effectively zero when it becomes negligible at the Objective Economic Scale relevant to the mechanism under analysis. Economic Zero therefore does not necessarily require the attainment of exact arithmetic zero.

Accordingly, conventional notation such as $z_n \to 0$ is used in this work only as a compact mathematical representation of finite economic diminution. It does not denote an asymptotic approximation to zero. The magnitude is reduced through realized finite changes until either it becomes negligible at the relevant Objective Economic Scale and is therefore treated as an Economic Zero, or exact arithmetic zero is attained or crossed where zero itself constitutes the relevant economic boundary. No indefinitely shrinking sequence of realized changes is implied.

The relevant question is whether the remaining magnitude continues to perform the economic function for which it enters the model. Once its magnitude has become negligible relative to the Objective Economic Scale of that mechanism, it may be treated as an **Economic Zero**.

This interpretation is not arbitrary. Whether a magnitude is economically negligible is determined by its causal role and by the Objective Economic Scale of the process under examination.

0.1. Consequences for Mathematical Representation

The preceding principles and postulates establish the relation between the economic model and the mathematical notation used throughout this work.

Remark 1 (Continuous Representation). Continuous-time notation remains admissible as an analytical representation of economic direction, relative movement, and movement toward an economically defined boundary or state. Expressions such as:

$$\dot{z}(t) > 0, \qquad \dot{z}(t) < 0, \qquad z(t) \to z^*$$

are analytical representations of direction and movement under the methodological principles established above. Their use does not alter the finite character of realized economic change; any asymptotic behavior must be supplied by an explicitly specified economic mechanism.

Continuous representation is useful when it makes the structure of a process more transparent. It may identify the direction of movement, compare two rates, characterize a functional relation, or represent repeated economic change toward an economically defined boundary or state.

Its analytical convenience, however, does not reverse the order established above:

$$\text{Economic Mechanism} \longrightarrow \text{Mathematical Representation.}$$

Corollary 1 (Boundary Attainment and Crossing)**.** *If an economically active mechanism progressively reduces a finite economic margin through realized finite changes at its Objective Economic Scale, the realized trajectory need not occupy the exact mathematical boundary as a separate economic state. It may either attain that boundary or cross it in a single finite transition.*

Let $M_n > 0$ denote a remaining finite margin before an economic boundary. A subsequent realized change may produce either $M_{n+1} = 0$ or $M_{n+1} < 0$.

The second case does not mean that the boundary was absent. It means that the finite economic transition crossed it.

Thus the economically relevant condition is:

$$M_n > 0, \qquad M_{n+1} \leq 0.$$

A mathematical equality defining a boundary and the realized economic transition across that boundary must therefore be distinguished.

Corollary 2 (No Autonomous Asymptotic Mechanism)**.** *Asymptotic behavior generated solely by a mathematical representation does not constitute an independent economic mechanism. If successive realized economic adjustments themselves become progressively smaller, the model must identify an economic mechanism that produces that reduction.*

The methodological structure of this work may therefore be summarized as:

$$\begin{gathered}\text{Economic Primacy over Mathematical Representation}\\ \Downarrow \\ \text{Finite Discrete Economic Change}\\ \text{Objective Economic Scale}\\ \text{Economic Finitude}\\ \Downarrow \\ \text{Economic Zero and Finite Economic Boundaries.}\end{gathered}$$

The economic analysis that follows is to be read under these methodological conditions. Continuous notation may be used where it clarifies the structure of an argument, but the substantive content of the argument remains determined by the finite economic mechanisms represented by that notation.

0.2. Notation of Economic Motion and Boundaries

The methodological principles established above require a notation that distinguishes an economic process from the particular mathematical representation used to describe it. The following symbols are therefore used throughout this work whenever the economic meaning of movement, exhaustion, or boundary crossing must be made explicit.

Definition 1 (Economic Transition)**.** The notation $X \xrightarrow{\mathrm{E}} Y$ denotes an **Economic Transition** from X toward Y.

An Economic Transition is a directional relation realized through finite economic changes at the Objective Economic Scale of the mechanism under analysis.

Thus $X \xrightarrow{\mathrm{E}} Y$ states that the economic mechanism represented by X generates movement toward the economic state or condition represented by Y.

The symbol differs from ordinary logical implication $X \implies Y$. The latter expresses a logical relation between propositions, whereas $X \xrightarrow{\mathrm{E}} Y$ expresses a realized economic direction or transition.

Definition 2 (Economic Direction)**.** The notation $z \uparrow_E$ denotes an economically realized upward movement of z, while $z \downarrow_E$ denotes an economically realized downward movement of z.

These symbols describe direction at the Objective Economic Scale without requiring equal increments between successive states.

A continuous representation may subsequently express the same directions as $\dot{z}(t) > 0$ or $\dot{z}(t) < 0$. These are analytical representations of $z \uparrow_E$ and $z \downarrow_E$, not independent economic assumptions.

Definition 3 (Economic Zero)**.** The symbol 0_E denotes the **state of Economic Zero**: the condition in which an economic magnitude has become negligible at the Objective Economic Scale relevant to the mechanism under analysis. It is an economically defined state, not a distinct arithmetic number.

Accordingly, $z \xrightarrow{\mathrm{E}} 0_E$ means that realized finite economic changes reduce z until it enters the state of Economic Zero.

The state of Economic Zero does not require exact arithmetic equality $z = 0$.

Thus conventional mathematical notation such as $z(t) \to 0$ may, where economically appropriate, represent $z \xrightarrow{\mathrm{E}} 0_E$.

The first is compact mathematical notation for the economic transition stated by the second.

Definition 4 (Transit Economic Boundary)**.** Let b denote either a finite numerical boundary or an economically defined boundary state, including 0_E. The notation $[b]_{\mathrm{C}}$ denotes a **Transit Economic Boundary**: a boundary that may be crossed by a finite realized economic change without terminating the underlying economic variable or process.

The notation $z \xrightarrow{\mathrm{E}} [b]_{\mathrm{C}} \xrightarrow{\mathrm{E}} \cdots$ means that z moves toward the boundary b, may attain or cross it, and may continue to move beyond it.

Because realized changes are finite, exact equality at the boundary is not required. A transition from $z_n > b$ to $z_{n+1} < b$ constitutes a crossing of $[b]_{\mathrm{C}}$. The boundary remains economically meaningful even though no realized state need satisfy $z_n = b$.

A particularly important case is arithmetic zero functioning as a Transit Economic Boundary:

$$z \xrightarrow{\mathrm{E}} [0]_{\mathrm{C}} \xrightarrow{\mathrm{E}} z < 0.$$

This notation means that z reaches or crosses zero through a finite realized change and continues into the economically meaningful region $z < 0$. A single transition may move the magnitude directly from $z_n > 0$ to $z_{n+1} < 0$.

Definition 5 (Terminal Economic Boundary)**.** The notation $[b]_{\mathrm{T}}$ denotes a **Terminal Economic Boundary**: a boundary at which the economic process or economic function under examination ceases to continue in the same form.

The notation $z \xrightarrow{\mathrm{E}} [b]_{\mathrm{T}}$ means that the economically relevant movement of z terminates when the boundary b is attained or crossed by a finite realized change.

A finite economic step may therefore be sufficiently large that its purely arithmetic continuation would pass beyond b. If states beyond b do not belong to the economic process being represented, the terminal economic state is nevertheless identified with $[b]_{\mathrm{T}}$.

Because the boundary value may itself be an economically defined state, the notation permits both $[0_E]_{\mathrm{C}}$ and $[0_E]_{\mathrm{T}}$.

In the first case, reaching Economic Zero does not terminate the broader economic process under examination. In the second case, reaching Economic Zero terminates the particular economic process or function for which that magnitude is relevant.

Thus a Terminal Economic Boundary differs fundamentally from a Transit Economic Boundary:

A Transit Economic Boundary $[b]_{\mathrm{C}}$ permits economically meaningful continuation after crossing, whereas attainment or crossing of a Terminal Economic Boundary $[b]_{\mathrm{T}}$ terminates the relevant economic process.

Definition 6 (Finite Economic Crossing)**.** The notation $a \bowtie_E b$ denotes a **Finite Economic Crossing** between two economic magnitudes.

It means that their ordering changes between successive realized economic states, even if exact equality is never itself realized.

For example, $a_n > b_n$ followed by $a_{n+1} \leq b_{n+1}$ is sufficient to establish $a \bowtie_E b$.

The crossing therefore does not require the existence of a realized state satisfying $a = b$.

This distinction is essential in a discrete economic process, because a finite transition may move directly from one side of a boundary to the other.

The notation introduced in this section may be summarized as:

$X \xrightarrow{\mathrm{E}} Y$	Economic Transition,
$z \uparrow_E$	upward Economic Direction,
$z \downarrow_E$	downward Economic Direction,
0_E	Economic Zero,
$[b]_{\mathrm{C}}$	Transit Economic Boundary,
$[b]_{\mathrm{T}}$	Terminal Economic Boundary,
$a \bowtie_E b$	Finite Economic Crossing.

These symbols identify the economic content of a dynamic argument directly. Continuous-time notation, limiting notation, and other mathematical representations may still be used where analytically useful, but their economic interpretation remains governed by the notation and methodological principles established in this chapter.

Part I.
The Pure Refrigerator Model

1. Idealized Gold

In this part, we begin with the simplest possible model. For the moment, the technical conditions are treated as given: technical progress, competition for labour, and the constraints of material reproduction will be introduced later. The first task is to define the object of analysis itself.

By **Gold** we shall mean money in the Marxian sense: a commodity functioning as the universal equivalent (Marx 1976, chap. 1, sec. 3). This does not necessarily refer to physical gold as a metal; the term *Gold* is used in order to distinguish commodity money from modern **Fiat Money**.

Idealized Gold is a money commodity that possesses its own value but is freed from an independent natural constraint on its reproduction.

It is defined by four properties.

Commodity Value. Idealized Gold remains a product of labour. Its production requires living labour and means of production, and it therefore possesses its own value. In this respect, it remains a commodity and is fundamentally distinct from pure Fiat Money.

Monetary Character. The commodity produced is simultaneously money. Its production is therefore not merely the production of another commodity that must subsequently be exchanged for money, but the direct production of the money commodity itself.

Unlimited Reproducibility. The production of Idealized Gold does not encounter an independent natural constraint analogous to the limited availability of deposits of Real Gold. Given labour and reproducible means of production, the output of the money commodity can be expanded in the same way as the output of ordinary industrial commodities.

Here, **Unlimited Reproducibility** does not denote infinite realized output. It means only that the model removes an independent natural scarcity boundary specific to the money commodity. At every realized economic state, production and output remain finite in accordance with the Postulate of Economic Finitude.

Monetary Persistence. Once produced and retained in monetary use, a unit of Idealized Gold continues to exist as a unit of the same money commodity. The model abstracts from physical loss and from non-monetary consumption of Gold unless such processes are explicitly introduced. Previously produced and newly produced units of Idealized Gold are therefore economically identical units of the same reproducible money commodity rather than separate historical vintages carrying different current values.

It is **Unlimited Reproducibility** that distinguishes Idealized Gold from Real Gold in the present thought experiment. Real Gold also possesses persistence, but its reproduction remains constrained by the independent natural conditions of mining: deposits, grades, discovery, location, and related material limits. Idealized Gold deliberately removes that independent natural scarcity constraint while retaining commodity value, monetary character, and monetary persistence.

We therefore consider a commodity for which four conditions hold simultaneously: commodity value, monetary function, free reproducibility, and monetary persistence.

This construction is not proposed as a description of an actually existing monetary system. It is a **Thought Experiment**. Its purpose is to determine whether commodity-money production can remain stable once the specific constraint associated with **Real Gold** is removed.

2. Gold Production and Monetary Adjustment

The peculiar position of gold within a commodity-money system was already central to Marx's analysis of money. Gold remains a commodity produced through labour while simultaneously occupying the position of the universal equivalent. Marx therefore treated the money commodity as distinct in its social function from ordinary commodities and, in *Capital*, Volume II, separately considered the reproduction of the money material and the specific position of the gold producer (Marx 1973, 1978).

The production of gold consequently raised a further question: how does the production of the money commodity respond to changes in its profitability? The idea that gold production contains an endogenous corrective tendency appeared in several forms in the literature. Fawcett, for example, connected increased profitability in gold mining with an expansion of gold production and a counteracting movement in the value of gold (Fawcett 1863). Related questions were discussed more broadly in the literature on gold production, monetary value, and prices (Bauer 1912; Gelderen 1912; Hilferding 1912; Varga 1911).

For the present argument, the most important formulation is that of Karl Kautsky.

Kautsky argued that when gold production yields an exceptional rate of profit, additional capital is attracted into gold production. This capital attracts additional labour-power, gold production expands, and the additional gold increases monetary demand for commodities. If the resulting movement of commodity prices is insufficient to remove the exceptional profitability of gold production, still more capital and labour are attracted into the sector. The movement continues until the extra profit disappears and the rate of profit in gold production is again brought into line with that of other branches (Kautsky 1913).

Gold production thus appears as part of an endogenous regulating mechanism: relative profitability redirects capital and labour, the production of the money commodity changes, and the initial differential is thereby corrected.

Yet Kautsky himself recognized that actual gold production does not take place under freely reproducible conditions. He emphasized the extraordinary heterogeneity and "lottery character" of gold mining: mines differ radically in productivity and profitability, richer fields may be discovered, the productivity of existing mines may decline, and sufficiently unprofitable mines are abandoned as capital leaves them (Kautsky 1913). Additional capital therefore cannot simply summon additional gold under unchanged productive conditions. Its result depends upon the particular natural conditions of the deposits available to it.

This leaves a simple question. What would become of Kautsky's regulating mechanism if it were granted conditions more favorable than those supplied by real gold itself? Suppose that the natural contingency of gold production were removed: additional capital could enter gold production and actually reproduce that production without encountering an independent limitation imposed by the particular deposits that happen to exist.

3. The Refrigerator Thought Experiment

The **Refrigerator Thought Experiment** is introduced in order to separate the monetary function from the natural scarcity of **Real Gold**.

Consider an ordinary reproducible commodity, such as a refrigerator. Its production does not depend on a unique deposit or on any other independent constraint. Given labour, capital, and reproducible means of production, the output of refrigerators can be expanded in the same way as the output of any other industrial commodity.

Now suppose that the refrigerator becomes a **Money Commodity**, that is, that it begins to perform the function of the universal equivalent.

Its productive nature does not change. It remains an ordinary reproducible commodity. Only its social function changes: the result of its production is no longer merely a commodity that must be sold for money, but money itself.

This is where the **Refrigerator Problem** arises: what happens to capitalist production if the money commodity can be reproduced as freely as an ordinary industrial commodity?

At this stage, technical progress, changes in the value of labour, differences in labour conditions, and the constraints of material reproduction are not considered.

4. Monetary Privilege

The main difference between **Idealized Gold** and an ordinary commodity is that the result of its production already possesses monetary form.

For ordinary commodity production, the process does not end with the creation of the product. After production, the commodity must still be realized on the market:

$$P \to C' \to M'$$

where P is the production process, C' is the commodity produced, and M' is the money obtained through its sale.

Until the transition

$$C' \to M'$$

has occurred, the value of capital has not yet been realized in monetary form.

The problem of this realization was the original point of departure of the broader work *The Forms of Gold*: how the monetary form of value relates to the realization of capital in an ordinary commodity economy, and what constraints arise from the structure of the money commodity itself.

In the case of **Idealized Gold**, this final stage is absent. If the commodity produced is itself money, then the result of production immediately assumes monetary form:

$$P \to M'$$

This is called **Direct Monetary Realization**.

This is what creates **Monetary Privilege**. An ordinary commodity must find a buyer in order to become money. A money commodity does not have to be sold as a distinct final product, because it already constitutes the universal form of value.

An important distinction follows. Ordinary production faces a **Final-Market Constraint**: expansion of output requires the possibility of realizing the additional commodity product.

For Idealized Gold, this constraint does not exist in the same form. Additional output does not require separate final demand for the money commodity itself as an object of consumption. The product produced already constitutes monetary wealth and can be stored, accumulated, or used as a means of circulation.

Monetary Privilege, however, concerns the form in which the product is realized. It does not exempt Idealized Gold from the determination of commodity value. A change in the conditions under which Gold is reproduced may therefore change the value represented by one physical unit of Gold even though the product itself requires no separate final sale.

This distinction between **Direct Monetary Realization** and the **Current Reproduction Value** of the money commodity becomes essential once technical differences are introduced.

5. Current Reproduction Value of Idealized Gold

Idealized Gold is both a reproducible commodity and money. Its monetary persistence therefore requires us to distinguish the physical date of production of a unit of Gold from its current economic value.

Let $p_{G,i}(t)$ denote the unit reproduction value of Idealized Gold under a realized Gold-production technique i at economic state t.

Previously produced and newly produced units are economically identical units of the same reproducible money commodity. Their current value is therefore governed by the **Current Reproduction Value** of Gold rather than by the historical conditions under which a particular unit was produced.

Within the idealized construction adopted in this work, we introduce the following model-specific regulating principle:

$$\boxed{p_G(t) = \min_{i \in \Theta_{\text{realized}}(t)} p_{G,i}(t).}$$

Here $\Theta_{\text{realized}}(t)$ denotes the set of Gold-production techniques that have actually been realized in production at state t.

The economic meaning is that once a lower-value reproduction technique has actually been realized, it becomes the regulating Current Reproduction Value of Gold.

This rule is a model-specific extension of current-reproduction-value reasoning under the joint assumptions of **Unlimited Reproducibility**, **Direct Monetary Realization**, and **Monetary Persistence**. It is not presented as a claim that Marx formulated this exact regulating equation.

If a newly realized reproduction condition reduces the unit value from p_G^0 to $p_G^1 < p_G^0$, then the Current Reproduction Value changes accordingly:

$$p_G^0 \longrightarrow p_G^1.$$

The unit devaluation is therefore:

$$\Delta p_G = p_G^1 - p_G^0 < 0.$$

For an individual capitalist holding a positive quantity of Gold $Q_{G,i} > 0$, this unit devaluation implies a loss in the current value of that individual holding:

$$\Delta W_{Q_{G,i}} = Q_{G,i} \Delta p_G < 0.$$

This private holding effect supplies the individual incentive relevant for Technical Selection. No aggregate Gold-stock value is required for the argument.

This is a **Devaluation of the Money Commodity**: the Current Reproduction Value of one unit of Gold has fallen.

The next chapters use this result to distinguish the technical incentives of ordinary commodity capital from those of Gold Capital.

6. Capital Selection Principle

After introducing **Monetary Privilege**, it is necessary to determine the criterion by which capital chooses between the production of Idealized Gold and the rest of the economy.

This criterion is the **Rate of Valorization**: the relative increase of advanced capital over the same comparable period of time.

Let g denote the **Gold Rate of Valorization**, and let $\bar{g}$ denote the **Non-Gold Rate of Valorization**.

By the **Non-Gold Economy** we mean the entire capitalist economy outside the production of Idealized Gold: the production of ordinary commodities, means of production, consumer goods, and other applications of capital.

The magnitude $\bar{g}$ expresses the comparable rate of valorization of capital in this remaining part of the economy. Both g and $\bar{g}$ are measured over the same period of time.

This is necessary because of differences in the **Turnover of Capital**. Ordinary capital may complete several production and realization cycles within a single period. Therefore, g and $\bar{g}$ already incorporate all differences in the speed of turnover and express the resulting valorization of capital over the same interval of time.

For Idealized Gold, the separate realization of its own product is absent: the commodity produced is already money. This difference is likewise already reflected in the resulting magnitude g.

If $g > \bar{g}$, the Capital Selection Principle gives the Economic Transition:

$$\boxed{g > \bar{g} \xrightarrow{\text{E}} \text{Capital Motion toward the Gold Sector.}}$$

If $g < \bar{g}$, then:

$$\boxed{g < \bar{g} \xrightarrow{\text{E}} \text{Capital Motion toward the Non-Gold Economy.}}$$

If $g = \bar{g}$, neither sector possesses a relative Valorization Advantage, and no directional Economic Transition follows from relative valorization alone.

This is the **Capital Selection Principle**: capital is distributed between the Gold Sector and the Non-Gold Economy according to their relative rates of valorization.

The quantity of already existing money does not directly determine this choice. As long as $g > \bar{g}$, the production of Idealized Gold remains the more profitable direction of capital accumulation.

7. Technical Selection

To determine the technical choice of the Gold Sector, it is first necessary to distinguish the technical incentive of ordinary commodity capital from the technical incentive of capital producing the money commodity.

At this stage, **Technical Progress** over time is absent. The current value conditions of production are treated as given. We compare technical choices that are available at the same realized economic state.

Let θ denote the **Technical Level** of an ordinary commodity-producing technique. A higher θ means greater physical productivity.

For the ordinary-commodity comparison, let $c(\theta)$ denote Constant Capital advanced under the technique, and let v denote Variable Capital advanced on labour power.

The Organic Composition of Capital may be represented by:

$$q(\theta) = \frac{c(\theta)}{v}.$$

Where more productive ordinary techniques are also more capital-intensive relative to living labour, we may have:

$$\frac{dq}{d\theta} > 0.$$

It is now necessary to specify the Rate of Valorization.

Let s_T denote the **Total Surplus Value over the Comparison Period**.

The period T is the same period over which the sectoral Rates of Valorization were compared in Chapter 6.

If one capital completes one turnover during this period while another completes several, then s_T includes all Surplus Value produced through all completed turnovers during T.

The Rate of Valorization over this period is therefore:

$$g_T = \frac{s_T}{c + v}.$$

Here, $c + v$ is advanced capital, while s_T is the total Surplus Value obtained by this capital during the comparable period.

For brevity, the index T may henceforth be omitted:

$$g = \frac{s}{c + v},$$

but s continues to denote total Surplus Value over the established comparison period rather than the Surplus Value of a single turnover.

Define:

$$e = \frac{s}{v}.$$

This is the **Period Rate of Surplus Value** relative to advanced Variable Capital.

Then:

$$g = \frac{e}{1+q}.$$

This identity describes the value composition of advanced capital and its relation to the Rate of Valorization. It does not by itself determine which technique an individual capital will select.

For ordinary commodity production, technical selection is governed by **Technical Competition**.

Greater physical productivity allows the same quantity of living labour, under otherwise comparable conditions, to produce a larger physical quantity of commodities. The Individual Value of one unit of output therefore falls.

Let $p_i(\theta)$ denote this Individual Value.

For a productivity improvement:

$$\frac{dp_i}{d\theta} < 0.$$

Let the prevailing Social Value of the ordinary commodity be $\bar{p}$.

If an individual capital introduces a more productive technique and obtains $p_i < \bar{p}$, it temporarily produces below the prevailing Social Value.

This creates the possibility of **Extra Profit**: the commodity may be realized at the existing Social Value, or the individual capitalist may reduce its selling price, expand market share, and displace less productive competitors.

Ordinary commodity capital therefore possesses a unilateral incentive to increase physical productivity even where the more productive method also requires a different value composition of capital.

A different technical logic operates in the Gold Sector.

Idealized Gold does not have to be transformed into money through a separate final-market realization. Moreover, Chapter 5 established that a lower realized unit reproduction value of Gold revalues identical existing and newly produced Gold at the new Current Reproduction Value.

Let ρ_G^0 denote the established physical productivity of the Gold-production process at the current realized economic state.

The present chapter considers a unilateral change in physical Gold productivity while the current input-value conditions and the value created by the normalized living-labour process are otherwise unchanged.

First consider a unilateral productivity increase $\rho_{G,i} > \rho_G^0$.

The same normalized living-labour process does not create additional New Value merely because it produces a larger physical quantity of Gold. With the transferred-value conditions of the process otherwise unchanged, the larger physical Gold output therefore possesses a lower Individual Reproduction Value per unit $p_{G,i} < p_G^0$.

From the Current Reproduction Value principle:

$$p_G = \min_{j \in \Theta_{\text{realized}}} p_{G,j},$$

the realization of this more productive Gold technique produces:

$$p_G^1 < p_G^0.$$

The Current Reproduction Value of each economically identical unit of Gold is consequently reduced.

For a Gold capitalist with a positive monetary holding $Q_{G,i} > 0$, this produces:

$$\Delta W_{Q_{G,i}} = Q_{G,i} \left(p_G^1 - p_G^0 \right) < 0.$$

At the same time, the additional physical Gold produced by the productivity increase does not create additional New Value merely by virtue of the larger physical output, and Gold Capital does not obtain the ordinary final-market Extra Profit mechanism described above.

Thus, under the stated conditions, unilateral improvement in physical Gold productivity creates no corresponding value gain from the larger physical output while it lowers the Current Reproduction Value of the money commodity and devalues the capitalist's positive Gold holding.

Now consider unilateral deterioration $\rho_{G,i} < \rho_G^0$.

Under otherwise unchanged value conditions, the Individual Reproduction Value of this less productive output is higher, $p_{G,i} > p_G^0$.

But the already realized Gold process with productivity ρ_G^0 continues to reproduce the same money commodity at p_G^0.

Therefore the Current Reproduction Value remains:

$$p_G = \min \left\{ p_G^0, p_{G,i} \right\} = p_G^0.$$

The individual capitalist cannot raise the Current Reproduction Value of Gold merely by producing the same money commodity less efficiently. The unilateral deterioration therefore yields no monetary-value advantage.

We consequently obtain the **Gold-Productivity Status Quo**:

$$\boxed{\rho_G = \rho_G^0.}$$

Under unchanged current value conditions, Gold Capital has no unilateral technical incentive to raise physical productivity above the established level and no unilateral advantage from reducing it below that level.

The Gold-Productivity Status Quo is a result for a fixed realized economic state. It does not imply that Gold productivity remains permanently constant over historical time.

If Technical Progress subsequently changes the value of the means of production used by the Gold Sector, the value conditions under which the technical choice is made also change. Gold Capital must then solve a new technical-selection problem.

That dynamic process is introduced in the next Part.

8. Extensive Expansion

The previous chapter established the **Gold-Productivity Status Quo** $\rho_G = \rho_G^0$ for the current realized economic state.

This does not prevent the Gold Sector from expanding its total scale of production.

The Gold Sector can expand through **Extensive Expansion**: an increase in the number of production complexes reproducing the same currently established Gold-production process.

This expansion is possible because of the already established properties of Idealized Gold: **Unlimited Reproducibility** and **Monetary Privilege**. A production complex can be reproduced whenever capital and labour are available, while the additional Gold output does not require a separate final market in order to become money.

Suppose that one production complex operating with the established Gold process requires k_G^0 units of advanced capital and l_G^0 units of living labour, and produces y_G^0 physical units of Idealized Gold during the comparable period.

If there are n identical production complexes, then:

$$K_G = nk_G^0, \qquad L_G = nl_G^0, \qquad Y_G = ny_G^0.$$

Here, n expresses the extensive scale of the Gold Sector under the currently established production process.

If $n \uparrow_E$, then:

$$K_G \uparrow_E, \qquad L_G \uparrow_E, \qquad Y_G \uparrow_E .$$

At the same time, the productivity of the replicated production process remains $\rho_G = \rho_G^0$.

It is therefore essential to distinguish:

$$\text{Extensive Growth of Total Gold Output} \neq \text{Increase in Gold Productivity.}$$

A larger total quantity of Gold produced by replicating an unchanged process does not by itself lower the unit Current Reproduction Value of Gold. Each additional production complex reproduces the same commodity under the same unit reproduction conditions.

If the Rate of Valorization of one such production complex is g, proportional replication of the same process preserves that Rate of Valorization while increasing the absolute mass of employed capital and produced Surplus Value.

Therefore, under $g > \bar{g}$, additional capital has an incentive to move into the Gold Sector.

In the pure model, the expansion of the Gold Sector therefore takes the form of successive finite increases in its extensive scale:

$$n \uparrow_E,$$

while the established Gold productivity remains unchanged:

$$\rho_G = \rho_G^0.$$

It is here that the basic content of the **Refrigerator Problem** already appears, although its dynamic technical development has not yet been introduced.

If $g > \bar{g}$, capital begins to leave the Non-Gold Economy and move into the Gold Sector.

Because Idealized Gold possesses **Direct Monetary Realization**, the additional Gold output generated by Extensive Expansion does not create a separate final-market realization constraint for the Gold product itself.

The pure model therefore distinguishes replication of an established Gold process from a change in the physical productivity of that process.

The first expands the total Gold Sector. The second changes the technical conditions under which one unit of Gold is reproduced and therefore requires the separate analysis developed in the next Part.

Part II. Technical Progress, Compensating Deterioration, and Gold Devaluation

9. Cheapening of Gold Constant Capital

Part I established the **Gold-Productivity Status Quo** $\rho_G = \rho_G^0$ for a fixed realized economic state.

Under unchanged current value conditions, an individual Gold capitalist has no unilateral incentive either to raise physical Gold productivity above the established level or to reduce it below that level.

We now introduce **Technical Progress** over time.

Technical Progress occurs not only inside a particular production process. It also changes the value of the reproducible means of production available to that process.

Tools, machines, materials, and other elements of Constant Capital are themselves reproducible commodities. When productivity rises in the industries producing them, their values fall.

To isolate this mechanism, let a_G^0 denote the physical bundle of means of production used by the currently established Gold-production process, and let $p_K(t)$ denote the value index of the reproducible means of production entering that bundle.

The value of Constant Capital required by the established process is therefore:

$$c_G(t) = a_G^0 p_K(t).$$

Technical Progress in the industries producing those means of production gives:

$$p_K(t) \downarrow_E .$$

For the currently established physical bundle $a_G^0 = \text{const}$, we therefore obtain:

$$c_G(t) \downarrow_E .$$

Throughout the operative Gold-production branch, $c_G(t)$ remains economically positive. The notation $c_G(t) \downarrow_E$ denotes realized cheapening of Gold Constant Capital and does not imply convergence to, or attainment of, arithmetic zero.

The economic meaning is that the same currently established Gold-production process can be reproduced with a smaller value of Constant Capital.

For the technical mechanism isolated in this Part, we adopt a normalized **circulating/full-use period abstraction**: the Constant Capital advanced for the representative Gold-production process over the comparison period is treated as transferred to the Gold output within that same period.

Accordingly, the same magnitude c_G enters both the reproduction-value accounting of the Gold output and the Rate of Valorization of the representative process.

This is a periodization assumption for the present mechanism. It does not claim that all actually existing productive capital is physically consumed within one period. Durable Fixed Capital could be represented by a separate depreciation and advance structure, but that extension is not required for the economic relation derived here.

Let N denote the New Value created by normalized living labour over the comparison period, and let ρ_G denote the physical quantity of Idealized Gold produced by the normalized Gold-production process during that period.

The Current Reproduction Value of one physical unit of Gold under that process is therefore:

$$p_G = \frac{c_G + N}{\rho_G}.$$

Consider an initial realized state:

$$c_{G,0}, \qquad \rho_{G,0}, \qquad p_G^0 = \frac{c_{G,0} + N}{\rho_{G,0}}.$$

Suppose Technical Progress subsequently cheapens the means of production required by the established Gold process, so that $c_{G,1} < c_{G,0}$.

If Gold Capital continued to use the same physical productivity $\rho_G = \rho_{G,0}$, then the reproduction value of Gold would become:

$$p_G^1 = \frac{c_{G,1} + N}{\rho_{G,0}}.$$

Since $c_{G,1} < c_{G,0}$, we obtain $p_G^1 < p_G^0$.

Thus, if no technical reaction occurred inside the Gold Sector, the technical cheapening of its Constant Capital would directly devalue the money commodity.

At the same time, the Rate of Valorization moves in the opposite direction.

Let v denote Variable Capital advanced over the same comparison period.

Surplus Value is $s = N - v$.

The Gold Rate of Valorization is therefore:

$$g_G = \frac{N - v}{c_G + v}.$$

At fixed N and v, we have:

$$\frac{\partial g_G}{\partial c_G} = -\frac{N - v}{(c_G + v)^2} < 0$$

as long as positive Surplus Value $N - v > 0$ exists.

Consequently:

$$c_G \downarrow_E \Longrightarrow g_G \uparrow_E .$$

Technical Progress therefore creates two simultaneous effects if Gold productivity initially remains unchanged:

$$c_G \downarrow_E \Longrightarrow \begin{cases} p_G \downarrow_E, \\ g_G \uparrow_E . \end{cases}$$

The Gold Sector therefore confronts a new Technical Selection problem that did not exist under the fixed value conditions of Part I.

10. Compensating Technical Deterioration

The sequence of economic events is essential.

Suppose the value of the means of production has already fallen, $c_{G,1} < c_{G,0}$, but no Gold has yet been produced under the newly cheapened condition.

The Current Reproduction Value of the already existing money commodity therefore remains $p_G = p_G^0$ at this realized state.

The cheapening of Constant Capital has occurred, but the lower reproduction value that would result from combining the new $c_{G,1}$ with the old productivity $\rho_{G,0}$ has not yet been realized in Gold production.

An individual Gold capitalist therefore chooses a production technique while facing $c_{G,1} < c_{G,0}$ and $p_G = p_G^0$.

At the relevant Objective Economic Scale, we abstract from the fine granularity of the technical set and suppose that Gold Capital can select among technically available production methods whose Constant-Capital value burden corresponds to the newly cheapened level $c_{G,1}$, but whose physical Gold productivities differ.

The compensating productivity is the productivity that exactly preserves the existing Current Reproduction Value of Gold.

It satisfies:

$$p_G^0 = \frac{c_{G,1} + N}{\rho_{G,1}^c}.$$

Therefore:

$$\rho_{G,1}^c = \frac{c_{G,1} + N}{p_G^0}.$$

Since:

$$p_G^0 = \frac{c_{G,0} + N}{\rho_{G,0}},$$

we may write:

$$\rho_{G,1}^c = \rho_{G,0} \frac{c_{G,1} + N}{c_{G,0} + N}.$$

Because $c_{G,1} < c_{G,0}$, we obtain $\rho_{G,1}^c < \rho_{G,0}$.

Thus the technique that preserves the Current Reproduction Value of Gold after the cheapening of Constant Capital is physically less productive than the previously established Gold technique.

We call this movement **Compensating Technical Deterioration**.

Its economic content can be seen from the individual capitalist's alternatives.

First, suppose the capitalist chooses a productivity above the compensating level, $\rho_{G,i} > \rho_{G,1}^c$.

Then:

$$p_{G,i} = \frac{c_{G,1} + N}{\rho_{G,i}} < p_G^0.$$

By the Current Reproduction Value principle established in Chapter 5, realization of this lower reproduction value reduces the Current Reproduction Value of the money commodity:

$$p_G \downarrow .$$

For a capitalist with a positive monetary Gold holding $Q_{G,i} > 0$, the value of that holding falls:

$$\Delta W_{Q_{G,i}} = Q_{G,i}\Delta p_G < 0.$$

The higher physical productivity does not create additional New Value merely by producing a larger physical quantity of Gold.

Moreover, at the same newly cheapened levels of $c_{G,1}$, N, and v, the Rate of Valorization remains:

$$g_{G,1} = \frac{N - v}{c_{G,1} + v}.$$

The higher physical Gold productivity therefore does not raise this Rate of Valorization.

It instead lowers the Current Reproduction Value of Gold and devalues the capitalist's positive monetary Gold holding.

Second, suppose the capitalist chooses exactly the compensating productivity $\rho_{G,i} = \rho^c_{G,1}$. Then $p_{G,i} = p^0_G$.

The Current Reproduction Value of Gold is preserved.

At the same time:

$$g_{G,1} = \frac{N - v}{c_{G,1} + v} > \frac{N - v}{c_{G,0} + v} = g_{G,0}.$$

Thus the capitalist obtains the increase in the Rate of Valorization generated by the cheapening of Constant Capital without producing a lower Current Reproduction Value of Gold.

Third, suppose the capitalist deteriorates productivity beyond the compensating level, $\rho_{G,i} < \rho^c_{G,1}$. Then $p_{G,i} > p^0_G$.

But the already existing reproduction conditions continue to establish the lower Current Reproduction Value p^0_G.

The individual capitalist cannot raise the Current Reproduction Value of the identical money commodity merely by producing it less efficiently.

The additional individual value expenditure is therefore not converted into a higher Current Reproduction Value of Gold.

Hence the relevant individual technical choice is the compensating boundary:

$$\rho^*_{G,i} = \rho^c_{G,1}.$$

The capitalist deteriorates physical Gold productivity only as far as is required to preserve the existing Current Reproduction Value.

The motive is individual.

No cartel, monopoly, explicit agreement, or expectation of an already completed collective technical change is required.

Each Gold capitalist confronts the same realized state $c_{G,1} < c_{G,0}$ and $p_G = p_G^0$, and independently faces the same technical-selection problem.

The limiting case $Q_{G,i} = 0$ is economically degenerate and is excluded from the present model. The operative case throughout is $Q_{G,i} > 0$.

When these individual choices are aggregated, the Gold Sector exhibits:

$$\boxed{c_G \downarrow_E, \qquad \rho_G \downarrow_E, \qquad p_G = \text{const}, \qquad g_G \uparrow_E,}$$

as long as a compensating lower-productivity technique remains economically available.

The causal structure must be kept distinct.

The increase in g_G is generated by $c_G \downarrow_E$. The decline in ρ_G does not itself create the higher Rate of Valorization.

Its function is to prevent the technical cheapening of Constant Capital from being translated into a lower Current Reproduction Value of Gold.

Technical Cheapening raises Gold Valorization, while Compensating Technical Deterioration preserves the Current Reproduction Value of Gold.

Because realized economic changes are finite, the set of technically available methods need not form a mathematically continuous scale.

A realized technical change may therefore undercompensate or overcompensate a particular arithmetic change in c_G.

The exact equation:

$$p_G^0 = \frac{c_G + N}{\rho_G}$$

represents the compensating technical relation at the relevant Objective Economic Scale.

It does not imply that actual production moves through infinitesimal technical adjustments.

The economically important result is the direction:

$$c_G \downarrow_E \xrightarrow{\text{E}} \rho_G \downarrow_E$$

while compensating deterioration remains technically possible.

11. Minimum Viable Gold Technique

Compensating Technical Deterioration cannot continue without a physical boundary.

Gold production requires a technically viable productive process.

There therefore exists a positive minimum physical productivity $\underline{\rho}_G > 0$, below which the process no longer constitutes viable production of Idealized Gold.

We call the production method realizing this lower physical boundary the **Minimum Viable Gold Technique**.

The Minimum Viable Gold Technique is the physical lower boundary of the compensating branch. It becomes relevant when Compensating Technical Deterioration reaches that boundary.

The compensating branch considered here remains within an economically positive Constant-Capital domain. Along the exact compensating relation:

$$p_G^0 = \frac{c_G + N}{\rho_G},$$

we have:

$$c_G = p_G^0 \rho_G - N.$$

The present model considers the economically relevant parameter domain in which the Minimum Viable Gold Technique becomes binding while Gold production still requires a positive Constant-Capital advance:

$$p_G^0 \underline{\rho}_G > N,$$

or equivalently:

$$\underline{\rho}_G > \frac{N}{p_G^0}.$$

Hence, when the Minimum Viable Gold Technique becomes binding:

$$c_G(T_{\min}) = p_G^0 \underline{\rho}_G - N > 0.$$

The compensating mechanism therefore reaches its physical productivity boundary before any arithmetic-zero condition for Gold Constant Capital could arise.

Instead, Technical Progress outside the Gold Sector repeatedly cheapens the Constant Capital required by Gold production, $c_G \downarrow_E$.

Each such realized cheapening creates a new compensating technical-selection problem.

While compensation remains possible:

$$c_G \downarrow_E \xrightarrow{\text{E}} \rho_G \downarrow_E .$$

The Gold Sector is therefore progressively driven toward the Minimum Viable Gold Technique through successive realized acts of Compensating Technical Deterioration.

The relevant lower physical boundary is $[\underline{\rho}_G]_{\text{T}}$.

Under continued technical cheapening, and in the absence of an independent mechanism arresting Compensating Technical Deterioration before this physical boundary becomes binding, the Economic Direction is:

$$\boxed{\rho_G \xrightarrow{\text{E}} [\underline{\rho}_G]_{\text{T}}.}$$

Under the finite-boundary convention established in Chapter 0, a final compensating selection may attain $\rho_G = \underline{\rho}_G$ or cross the arithmetic minimum $\rho_G < \underline{\rho}_G$.

Since the latter does not constitute viable Gold production, the compensating process terminates at $[\underline{\rho}_G]_{\text{T}}$.

Let $T_{\min}$ denote the first realized economic state at which the Minimum Viable Gold Technique becomes binding. Thus $\rho_G(T_{\min}) = [\underline{\rho}_G]_{\text{T}}$.

At $T_{\min}$, the internal technical mechanism by which Gold Capital has compensated for the cheapening of its Constant Capital is exhausted. No further economically viable decline in ρ_G is available as a Gold-production response.

12. Technical Devaluation beyond the Minimum Technique

Suppose the Minimum Viable Gold Technique has become binding, $\rho_G = \underline{\rho}_G$.

Compensating Technical Deterioration is now exhausted.

If Technical Progress continues to cheapen the Constant Capital entering Gold production, $c_G \downarrow_E$, Gold Capital can no longer preserve the previous Current Reproduction Value by reducing physical productivity.

The Current Reproduction Value is now:

$$p_G = \frac{c_G + N}{\underline{\rho}_G}.$$

Therefore:

$$c_G \downarrow_E \Longrightarrow p_G \downarrow_E .$$

At the same time, for fixed N and v, the Gold Rate of Valorization remains:

$$g_G = \frac{N - v}{c_G + v}.$$

Hence:

$$c_G \downarrow_E \Longrightarrow g_G \uparrow_E .$$

Beyond the Minimum Viable Gold Technique, Technical Progress therefore generates the simultaneous movement:

$$p_G \downarrow_E, \qquad g_G \uparrow_E .$$

This is the **Technical Devaluation of the Money Commodity**.

The production of Idealized Gold may become increasingly attractive to capital in terms of its Rate of Valorization while the Current Reproduction Value represented by each physical unit of Gold moves downward.

Because Gold possesses Monetary Persistence, the lower Current Reproduction Value does not apply only to newly produced Gold.

Previously produced and newly produced units remain economically identical units of the same money commodity.

Therefore $p_G \downarrow_E$ means that the Current Reproduction Value represented by each unit of Idealized Gold falls.

Repeated technical cheapening beyond $[\underline{\rho}_G]_{\mathrm{T}}$ therefore produces repeated unit devaluation of Idealized Gold.

This result does not by itself establish the immediate disappearance of the monetary function of Gold.

A monetary system may, for some economically realized interval, continue to employ a money commodity whose Current Reproduction Value is changing.

However, recurrent and expected Technical Devaluation creates a distinct monetary problem: continued holding of Idealized Gold as monetary wealth becomes exposed to repeated reductions in Current Reproduction Value.

Whether this process becomes sufficient to terminate the monetary function of Idealized Gold, or whether Idealized Gold continues to function as money while the other Refrigerator mechanisms proceed, will be considered in the final analysis of the model.

For the present Part, the necessary result is:

$$\boxed{\rho_G \xrightarrow{\mathrm{E}} [\underline{\rho}_G]_{\mathrm{T}} \quad \Longrightarrow \quad \text{further } c_G \downarrow_E \Longrightarrow p_G \downarrow_E .}$$

Thus the Minimum Viable Gold Technique is the terminal boundary of the internal compensating technical mechanism, not necessarily the terminal state of the entire economic model.

13. Cheapening of Labour Power

Technical Progress cheapens not only the means of production used by the Gold Sector.

It also reduces the value of the commodities required for the reproduction of labour power.

To separate this mechanism from **Labour Competition**, which will be introduced later, let $v^0(t)$ denote the **Baseline Value of Labour Power**.

It represents the value of the means of subsistence required for the reproduction of labour power under the given social conditions, before the emergence of additional pressure generated by competition from the Gold Sector for workers.

Let the bundle of consumption goods required for the worker be denoted by B. Let their total value at time t be $P_B(t)$. The Baseline Value of Labour Power is therefore $v^0(t) = P_B(t)$.

Technical Progress in the industries producing the necessary means of subsistence raises their productivity and lowers their value, $P_B(t) \downarrow_E$. Consequently, $v^0(t) \downarrow_E$.

This process occurs before the introduction of Labour Competition.

Here, the decline in v^0 is caused exclusively by the cheapening of the necessary product.

Let N denote the New Value created by normalized living labour over the comparable period. At this stage $N = \text{const}$.

Baseline Surplus Value is therefore:

$$s^0(t) = N - v^0(t).$$

Hence $v^0(t) \downarrow_E$ produces $s^0(t) \uparrow_E$.

It is important to distinguish this mechanism from the cheapening of Gold Constant Capital.

Since:

$$v^0 + s^0 = N,$$

the cheapening of labour power redistributes the fixed New Value between Variable Capital and Surplus Value.

At fixed c_G, ρ_G, and N, the total value entering the Gold output remains $c_G + N$.

Therefore the isolated decline:

$$v^0 \downarrow_E$$

does not itself lower the Current Reproduction Value of Gold:

$$p_G = \frac{c_G + N}{\rho_G}.$$

Thus:

$$v^0 \downarrow_E \not\Longrightarrow p_G \downarrow_E$$

when c_G, ρ_G, and N are held fixed.

Its direct effect is instead on Surplus Value and the Rate of Valorization.

The Baseline Rate of Valorization of the Gold Sector is:

$$g^0(t) = \frac{N - v^0(t)}{c_G(t) + v^0(t)}.$$

At fixed c_G, we have:

$$\frac{\partial g^0}{\partial v^0} = -\frac{c_G + N}{\left(c_G + v^0\right)^2} < 0.$$

Therefore:

$$v^0(t) \downarrow_E \Longrightarrow g^0(t) \uparrow_E .$$

Chapter 9 established independently that:

$$c_G(t) \downarrow_E .$$

At positive Baseline Surplus Value $N - v^0 > 0$, we also have:

$$\frac{\partial g^0}{\partial c_G} = -\frac{N - v^0}{\left(c_G + v^0\right)^2} < 0.$$

Hence:

$$c_G(t) \downarrow_E \Longrightarrow g^0(t) \uparrow_E .$$

General Technical Progress therefore acts through two distinct channels, $c_G(t) \downarrow_E$ and $v^0(t) \downarrow_E$, both of which raise $g^0(t)$.

The channels nevertheless have different value effects.

The decline in v^0 raises Surplus Value by changing the division $N = v^0 + s^0$ without by itself changing $c_G + N$. The decline in c_G directly lowers the value burden of the Gold-production process.

Before the Minimum Viable Gold Technique becomes binding, Compensating Technical Deterioration can offset the effect of $c_G \downarrow_E$ on the unit Current Reproduction Value by producing $\rho_G \downarrow_E$.

Thus, on the compensating branch:

$$c_G \downarrow_E, \qquad v^0 \downarrow_E, \qquad \rho_G \downarrow_E,$$

while p_G is preserved at the relevant Objective Economic Scale and:

$$g^0(t) \uparrow_E .$$

After:

$$\rho_G \xrightarrow{\mathrm{E}} [\underline{\rho}_G]_{\mathrm{T}},$$

the compensating technical channel is exhausted.

Further $c_G \downarrow_E$ then reduces p_G, while both the cheapening of Constant Capital and the cheapening of labour power continue, where economically realized, to raise the Baseline Rate of Valorization.

The magnitude $g^0(t)$ therefore continues to express the **Baseline Rate of Valorization** of the Gold Sector before the introduction of the separate mechanism of Labour Competition.

The resulting economic direction is:

$$g^0(t) \uparrow_E .$$

General Technical Progress thus strengthens the Gold Sector's Baseline Rate of Valorization through both the cheapening of Constant Capital and the cheapening of labour power, while only the former creates the technical-selection problem that produces Compensating Technical Deterioration and, after its physical exhaustion, Technical Devaluation of the money commodity.

Part III. Ordinary Capital and the Profit-Rate Crossing

14. Technical Competition and the Composition of Non-Gold Capital

Chapter 7 established the technical-selection logic of ordinary commodity capital.

Unlike Idealized Gold, an ordinary commodity must be realized as a commodity on a market. A more productive individual capital can temporarily produce below the prevailing Social Value, obtain **Extra Profit**, reduce its selling price, expand its market share, and displace less productive competitors.

Technical Competition therefore gives ordinary commodity capital a persistent incentive to introduce more productive methods of production.

We now consider the corresponding technical tendency of the entire **Non-Gold Economy** over time.

Let $\bar{c}(t)$ denote average Constant Capital, and let $\bar{v}(t)$ denote average Variable Capital in the Non-Gold Economy over the common comparison period.

Define the value-composition index:

$$\bar{q}(t) = \frac{\bar{c}(t)}{\bar{v}(t)}.$$

In Marxian terminology, the Organic Composition of Capital refers to the value composition insofar as it reflects the underlying technical composition of production.

In this work, $\bar{q}(t)$ is used as the value-composition representation of the technical tendency generated by Technical Competition in the Non-Gold Economy.

More productive capitalist methods generally expand the role of means of production relative to living labour.

For the Non-Gold Economy, we therefore adopt the Marxian technical tendency:

$$\bar{q}(t) \uparrow_E .$$

This tendency is important, but it must not be interpreted as the sole determinant of the Non-Gold Rate of Valorization.

Let $\bar{s}$ denote Total Surplus Value produced by the Non-Gold Economy over the same comparable period used throughout the model.

Define:

$$\bar{e} = \frac{\bar{s}}{\bar{v}}.$$

The Non-Gold Rate of Valorization is:

$$\bar{g}(t) = \frac{\bar{s}(t)}{\bar{c}(t) + \bar{v}(t)}.$$

Equivalently:

$$\bar{g}(t) = \frac{\bar{e}(t)}{1 + \bar{q}(t)}.$$

Other things being equal, $\bar{q}(t) \uparrow_E$ creates downward pressure on $\bar{g}(t)$.

But the qualification *other things being equal* is essential.

Technical Progress may simultaneously cheapen labour power, alter the Rate of Surplus Value, accelerate the Turnover of Capital, and change other conditions affecting the total Surplus Value produced by a given advanced capital during the common comparison period.

Accordingly, the present chapter does not attempt to derive the complete movement of $\bar{g}(t)$ from $\bar{q}(t)$ alone.

The tendency $\bar{q}(t) \uparrow_E$ is retained as the technical-composition background of Non-Gold capitalist development.

15. Falling Rate of Valorization

A rising composition of Non-Gold Capital creates downward pressure on the Rate of Valorization but does not by itself determine its complete movement. The present model therefore adopts the following Economic Direction.

The magnitude $\bar{g}(t)$ denotes the **Non-Gold Rate of Valorization** over the common comparable period.

It includes all Surplus Value $\bar{s}_T$ obtained by the corresponding advanced capital during that period, including the effects of all completed turnovers.

An acceleration of the **Turnover of Capital** is therefore already reflected in $\bar{g}(t)$. If faster turnover permits the same advanced capital to produce a greater total Surplus Value during the comparison period, that effect enters $\bar{s}_T$ and consequently enters the measured Rate of Valorization.

Thus the model does not compare the profit of one isolated turnover.

It compares the complete realized capacity of capital for valorization over the same period of time.

Within this work, the Marxian **Tendency of the Rate of Profit to Fall** is adopted and applied to the Rate of Valorization of the Non-Gold Economy defined here (Marx 1981, pt. III, chap. 13).

Accordingly, we assume the Economic Direction:

$$\boxed{\bar{g}(t) \downarrow_E .}$$

This means that, with the development of capitalist production, the resulting comparable Rate of Valorization of Non-Gold Capital declines through successive realized economic changes.

The notation $\bar{g}(t) \downarrow_E$ specifies an Economic Direction.

It does not imply asymptotic approximation to an independently specified numerical limit.

This proposition is used as a Marxian assumption concerning the general dynamics of the Non-Gold Economy.

A separate derivation of the Tendency of the Rate of Profit to Fall and a complete analysis of all **Counteracting Factors** are beyond the scope of the present work.

16. Profit-Rate Crossing

We can now compare the economic directions of the Gold Sector and the Non-Gold Economy.

For the Gold Sector, Chapter 13 defined the **Baseline Rate of Valorization**:

$$g^0(t) = \frac{N - v^0(t)}{c_G(t) + v^0(t)}.$$

The preceding Part established $c_G(t) \downarrow_E$ and $v^0(t) \downarrow_E$, both of which raise $g^0(t)$.

At positive Baseline Surplus Value $N - v^0 > 0$, we have:

$$\frac{\partial g^0}{\partial c_G} = -\frac{N - v^0}{\left(c_G + v^0\right)^2} < 0,$$

and:

$$\frac{\partial g^0}{\partial v^0} = -\frac{c_G + N}{\left(c_G + v^0\right)^2} < 0.$$

Therefore:

$$c_G(t) \downarrow_E, \qquad v^0(t) \downarrow_E \Longrightarrow g^0(t) \uparrow_E .$$

The movement of physical Gold productivity does not reverse this result.

Before the Minimum Viable Gold Technique becomes binding, Compensating Technical Deterioration produces $\rho_G(t) \downarrow_E$ in response to $c_G(t) \downarrow_E$.

Its function is to preserve the Current Reproduction Value of Gold at the relevant Objective Economic Scale.

It does not restore the previous value of advanced Constant Capital.

Thus, on the compensating branch:

$$c_G(t) \downarrow_E, \qquad v^0(t) \downarrow_E, \qquad \rho_G(t) \downarrow_E,$$

while:

$$g^0(t) \uparrow_E .$$

If the Minimum Viable Gold Technique becomes binding:

$$\rho_G \xrightarrow{\text{E}} [\underline{\rho}_G]_{\text{T}},$$

further Technical Progress may instead produce:

$$p_G(t) \downarrow_E .$$

This Technical Devaluation of the money commodity does not, by itself, reverse the movement of the Gold Rate of Valorization.

As long as Idealized Gold continues to function as the money commodity of the model and:

$$c_G(t) \downarrow_E, \qquad v^0(t) \downarrow_E,$$

the Baseline Rate of Valorization continues to have the Economic Direction:

$$g^0(t) \uparrow_E .$$

Unless explicitly stated otherwise, the subsequent internal dynamics are analyzed on the branch in which Idealized Gold remains economically operative as money. Abandonment of Idealized Gold terminates this branch and is treated separately in the final analysis of the model.

For the Non-Gold Economy, the previous chapter adopted $\bar{g}(t) \downarrow_E$.

The two Rates of Valorization therefore possess opposite Economic Directions:

$$g^0(t) \uparrow_E, \qquad \bar{g}(t) \downarrow_E .$$

If the Gold Sector initially already satisfies $g_0^0 > \bar{g}_0$, its Valorization Advantage exists from the beginning.

The more important case is $g_0^0 < \bar{g}_0$. Take an initial realized state with $0 < g_0^0 < \bar{g}_0$, where g_0^0 is economically positive at the relevant Objective Economic Scale.

Define the remaining **Valorization Gap**:

$$H(t) = \bar{g}(t) - g^0(t).$$

At the initial state, $H_0 > 0$. Since $g^0(t) \uparrow_E$ while $\bar{g}(t) \downarrow_E$, the positive Valorization Gap moves as $H(t) \downarrow_E$.

No positive Terminal Economic Boundary for H has been introduced. Under the finite-crossing convention established in Subsection 0.2, continued realized movement therefore attains or crosses the zero-gap boundary:

$$H(t) \xrightarrow{\mathrm{E}} [0]_{\mathrm{C}}.$$

Because $H = \bar{g} - g^0$, attainment gives $g^0 = \bar{g}$, while crossing gives $g^0 > \bar{g}$. Hence:

$$\boxed{g^0 \bowtie_E \bar{g}.}$$

Equivalently, the crossing is realized whenever two successive economic states satisfy:

$$g_j^0 < \bar{g}_j, \qquad g_{j+1}^0 \geq \bar{g}_{j+1}.$$

The second relation may be either equality or strict inequality.

In the stronger case in which the Non-Gold Rate of Valorization reaches Economic Zero:

$$\bar{g} \xrightarrow{\mathrm{E}} 0_E,$$

the preceding crossing condition is necessarily satisfied provided that an economically positive Gold Rate of Valorization has already been realized on the same comparable scale.

The Profit-Rate Crossing is therefore identified by the finite reversal of the ordering of the two economically comparable Rates of Valorization.

16.1. Normalized Technical Ordering after the Crossing

The Labour Competition mechanism developed in the next Part requires a comparison of the value-capital burden associated with the same normalized unit of living labour in the two sectors.

We therefore now specialize the intersectoral comparison to a common normalized production unit.

Let N denote the New Value created by that normalized living labour over the common comparison period.

Before additional Labour Competition begins, let the Baseline Value of Labour Power be common to the two sectors:

$$v_G^0 = v_A^0 = v^0.$$

Let c_G denote the Constant Capital advanced alongside this normalized unit of labour in the Gold Sector, and let c_A denote the corresponding average Constant-Capital burden of the normalized Non-Gold production unit used for the intersectoral comparison.

The normalization is chosen so that the sectoral Rates of Valorization over the established common comparison period are represented by:

$$g^0 = \frac{N - v^0}{c_G + v^0},$$

and:

$$\bar{g} = \frac{N - v^0}{c_A + v^0}.$$

This representation makes explicit the technical comparison that will be used in the subsequent Labour Competition analysis.

It does not replace the earlier aggregate definition of the Non-Gold Rate of Valorization; it is its normalized representative form for the intersectoral capital-and-labour comparison developed from this point onward.

As long as positive Baseline Surplus Value $N - v^0 > 0$ exists, the difference between the two normalized Rates of Valorization is:

$$g^0 - \bar{g} = \frac{\left(N - v^0\right)\left(c_A - c_G\right)}{\left(c_G + v^0\right)\left(c_A + v^0\right)}.$$

Therefore:

$$g^0 > \bar{g} \iff c_G < c_A.$$

Similarly:

$$g^0 = \bar{g} \iff c_G = c_A,$$

and:

$$g^0 < \bar{g} \iff c_G > c_A.$$

Thus, within the normalized intersectoral representation used for the subsequent Labour Competition mechanism, the Profit-Rate Crossing carries the corresponding change in Constant-Capital ordering: exact equality $g^0 = \bar{g}$ corresponds to $c_G = c_A$, while a direct crossing from $g^0 < \bar{g}$ to $g^0 > \bar{g}$ corresponds to a direct change from $c_G > c_A$ to $c_G < c_A$.

The inequality $c_G < c_A$ follows directly from the realized post-crossing Valorization Advantage within the normalized intersectoral comparison.

The Capital Selection Principle then gives:

$$g^0 > \bar{g} \xrightarrow{\text{E}} \text{Capital Motion toward the Gold Sector.}$$

The adopted economic dynamics therefore generate the Refrigerator Problem in the post-crossing state.

The next Part analyzes the labour-market consequences of this post-crossing state:

$$g^0 > \bar{g}, \qquad c_G < c_A.$$

Part IV.
Labour Competition

17. Total Labour Cost

The preceding Part established the post-crossing state:

$$g^0(t) > \bar{g}(t).$$

On this branch, the Capital Selection Principle directs capital toward the Gold Sector.

As capital begins to move toward the Gold Sector, Gold production expands and requires additional labour.

It is therefore necessary to determine the costs that capital must bear in order to attract and reproduce labour power.

Let w denote the **Money Wage**.

The Money Wage does not exhaust all expenditures that capital may incur in order to obtain and reproduce labour power.

Capital may also provide housing, food, transport, insurance, medical care, and other forms of compensation whose value is borne by capital.

Let a denote the monetary equivalent of these additional expenditures. We define $\ell = w + a$, called the **Total Labour Cost**.

It expresses the total value advanced by capital in order to obtain and reproduce one unit of normalized labour, regardless of the particular form taken by that expenditure.

For capital, an additional wage payment and a non-wage benefit of equivalent value both increase the value advanced for labour.

They are therefore combined in ℓ.

Chapter 13 introduced $v^0(t)$, the **Baseline Value of Labour Power** determined by the value of the necessary product before the additional pressure generated by Gold-Sector competition for workers.

From this point onward, ℓ denotes the actual labour cost borne by capital after competitive labour-market pressure is introduced.

Before this additional competition begins, the normalized labour cost may be taken as $\ell = v^0$. As the Gold Sector expands and competes for labour, actual labour costs may rise above the baseline, $\ell > v^0$.

The distinction between $v^0(t)$ and $\ell(t)$ therefore separates two different mechanisms.

The first is the technical cheapening of the reproduction of labour power:

$$v^0(t) \downarrow_E .$$

The second is the competitive increase in the actual value advanced for labour:

$$\ell(t) \uparrow_E .$$

These movements may operate simultaneously and in opposite directions.

18. Labour Competition

The post-crossing ordering $c_G < c_A$ is the starting point for the labour-demand comparison. Here c_G and c_A denote the Constant-Capital burdens per normalized unit of living labour. The comparison does not require equal physical output per unit of labour and is unaffected by changes in Gold productivity ρ_G generated by Compensating Technical Deterioration.

Before the emergence of additional Labour Competition, normalized labour power bears the same Baseline Value in the two sectors, $\ell_G = \ell_A = v^0$.

Labour power is mobile between the Gold Sector and the Non-Gold Economy.

Let ℓ_G denote the **Total Labour Cost** in the Gold Sector, and let ℓ_A denote the Total Labour Cost in the Non-Gold Economy.

Consider a finite realized reallocation of capital $\Delta K > 0$ from the Non-Gold Economy into the Gold Sector.

At the initial common Total Labour Cost ℓ, the contraction of capital in the Non-Gold Economy releases:

$$\Delta L_A^- = \frac{\Delta K}{c_A + \ell}$$

units of normalized labour.

The same finite quantity of capital, after moving into the Gold Sector, creates demand for:

$$\Delta L_G^+ = \frac{\Delta K}{c_G + \ell}$$

units of normalized labour.

Since:

$$c_G < c_A,$$

we have:

$$c_G + \ell < c_A + \ell,$$

and therefore:

$$\frac{1}{c_G + \ell} > \frac{1}{c_A + \ell}.$$

Hence:

$$\Delta L_G^+ > \Delta L_A^-.$$

The finite net change in labour demand is:

$$\Delta L^d = \Delta K\left(\frac{1}{c_G + \ell} - \frac{1}{c_A + \ell}\right) > 0.$$

Thus the same finite quantity of capital requires more normalized labour after moving into the Gold Sector than it releases by leaving the Non-Gold Economy.

Given the currently unchanged total supply of available labour, this reallocation creates additional demand for labour power.

The Gold Sector must therefore attract workers who would otherwise remain employed in the Non-Gold Economy.

This produces upward pressure on its Total Labour Cost, $\ell_G > v^0$.

This constitutes **Labour Competition**.

As ℓ_G rises, Surplus Value in the Gold Sector is $s_G = N - \ell_G$. Therefore $\ell_G \uparrow_E$ produces $s_G \downarrow_E$.

The Gold Rate of Valorization becomes:

$$g = \frac{N - \ell_G}{c_G + \ell_G}.$$

At fixed N and c_G, we have:

$$\frac{\partial g}{\partial \ell_G} = -\frac{c_G + N}{(c_G + \ell_G)^2} < 0.$$

Hence:

$$\ell_G \uparrow_E \Longrightarrow g \downarrow_E .$$

This is the first internal labour-market reaction generated by Goldization itself.

The Gold Sector's expansion raises the demand for labour, and the resulting increase in its Total Labour Cost reduces its Rate of Valorization.

Whether this reaction can restore a stable positive equality between the two sectoral Rates of Valorization requires us to consider the response of the Non-Gold Economy.

19. Economy-Wide Compensation Adjustment

The additional demand for labour generated by capital reallocation into the Gold Sector initially raises ℓ_G above ℓ_A.

Because labour power is mobile between the two sectors, this pressure cannot be treated as permanently confined to Gold production.

To retain labour power, Non-Gold Capital must also respond to the higher compensation available in the Gold Sector.

The common labour market therefore generates finite realized adjustments toward a common **Competitive Total Labour Cost**:

$$\ell_G \xrightarrow{\mathrm{E}} \ell, \qquad \ell_A \xrightarrow{\mathrm{E}} \ell.$$

Here ℓ denotes the competitively realized Total Labour Cost common to the normalized intersectoral comparison.

The Economic Transition notation $\xrightarrow{\mathrm{E}}$ does not imply asymptotic convergence.

It states that labour-market competition produces realized finite adjustments toward a common economically operative labour-cost level.

For the same normalized unit of living labour, the Gold Rate of Valorization is then:

$$g = \frac{N - \ell}{c_G + \ell},$$

while the Non-Gold Rate of Valorization is:

$$\bar{g} = \frac{N - \ell}{c_A + \ell}.$$

An increase in ℓ acts on both Rates of Valorization through two channels. It reduces the remaining Surplus Value $N - \ell$ and increases the labour cost advanced in the denominator.

For the Gold Sector:

$$\frac{\partial g}{\partial \ell} = -\frac{c_G + N}{(c_G + \ell)^2} < 0.$$

For the Non-Gold Economy:

$$\frac{\partial \bar{g}}{\partial \ell} = -\frac{c_A + N}{(c_A + \ell)^2} < 0.$$

Therefore:

$$\ell \uparrow_E \Longrightarrow g \downarrow_E, \qquad \bar{g} \downarrow_E .$$

The labour-market pressure generated initially by Goldization therefore spreads through the common labour market to the economy as a whole.

This result does not yet determine whether the two Rates of Valorization become equal.

That question depends on whether a common increase in Total Labour Cost removes the technical ordering:

$$c_G < c_A$$

from the relative Rate-of-Valorization comparison.

20. Failure of Labour-Cost Equalization

With labour-market competition producing a common Competitive Total Labour Cost ℓ, the remaining question is whether this economy-wide adjustment eliminates the Gold Sector's Valorization Advantage.

The comparison is made at a realized technical state on the post-crossing branch for which:

$$c_G < c_A.$$

The magnitudes c_G and c_A need not remain numerically constant over historical time.

The result derived here is statewise: at every realized state in which the post-crossing technical ordering:

$$c_G < c_A$$

holds, we determine whether a common labour-cost increase by itself can eliminate the Gold Sector's relative advantage.

For an identical normalized unit of living labour, New Value is the same in the two sectors, $N_G = N_A = N$.

After Total Labour Costs have been competitively equalized, the Gold Rate of Valorization is:

$$g = \frac{N - \ell}{c_G + \ell},$$

while the Non-Gold Rate of Valorization is:

$$\bar{g} = \frac{N - \ell}{c_A + \ell}.$$

Their difference is:

$$g - \bar{g} = \frac{(N - \ell)(c_A - c_G)}{(c_G + \ell)(c_A + \ell)}.$$

As long as positive Surplus Value $N - \ell > 0$ exists and the post-crossing technical ordering $c_A - c_G > 0$ remains, all factors in the denominator are positive, so $g - \bar{g} > 0$, hence $g > \bar{g}$.

As long as $N - \ell > 0$ and $c_G < c_A$, a common increase in Total Labour Cost reduces both Rates of Valorization but does not reverse their ordering or generate a stable positive equality through economy-wide labour-cost equalization alone.

For exact arithmetic equality $g = \bar{g}$ under $c_G < c_A$, the common numerator must disappear: $N - \ell = 0$. Therefore $\ell = N$, so $s = N - \ell = 0$ and consequently $g = \bar{g} = 0$.

Thus exact equality generated by common labour-cost adjustment alone is obtained only by eliminating positive Surplus Value itself.

This exact algebraic result must be distinguished from the later dynamic Terminal Economic Boundary.

Under the methodological principles of this work, positive Surplus Value need not move asymptotically to arithmetic zero or occupy:

$$N - \ell = 0$$

as a separate realized state.

A finite economic adjustment may instead bring the remaining Surplus Value to Economic Zero or cross the positive-Surplus-Value boundary directly.

Economy-wide Labour-Cost Equalization alone therefore cannot produce a stable positive equality of g and $\bar{g}$.

This conclusion concerns economy-wide labour-cost adjustment alone.

It does not exclude the possibility of a separate sector-specific burden borne only by the Gold Sector.

The next Part introduces precisely such a stronger counteracting mechanism through **Structural Labour Disadvantage**.

Part V.
Structural Labour Disadvantage

This Part determines whether a sector-specific labour disadvantage can provide a stable positive resolution of the Refrigerator Problem.

21. Compensating Differential

Until now, after accounting for **Total Labour Cost**, labour conditions in the Gold Sector and the Non-Gold Economy have been treated as equivalent.

We now introduce **Structural Labour Disadvantage**: an objective difference in labour conditions arising from the character of production itself.

For example, Gold production may require underground work, remote locations, or other conditions whose compensation requires additional expenditure by capital.

Let d denote the **Compensating Differential**: the additional value that Gold Capital must spend per normalized unit of labour if such structurally worse labour conditions are compensated.

The introduction of d does not imply that capitalist labour markets necessarily compensate such disadvantages in practice.

It is introduced as a deliberately strong sector-specific counteracting mechanism in order to determine whether an additional labour-cost burden borne specifically by the Gold Sector can neutralize its Valorization Advantage.

If no such compensation is provided, the corresponding case is simply $d = 0$.

Let the common Competitive Total Labour Cost in the normalized intersectoral comparison be ℓ. The actual labour-related value burden borne by the Gold Sector is then $\ell + d$.

The Gold Rate of Valorization becomes:

$$g = \frac{N - \ell - d}{c_G + \ell + d}.$$

For the Non-Gold Economy:

$$\bar{g} = \frac{N - \ell}{c_A + \ell}.$$

Here d affects the Gold Sector in two ways simultaneously.

It increases the value advanced by Gold Capital and reduces the remaining Surplus Value:

$$s_G = N - \ell - d.$$

The preceding Parts established the post-crossing normalized technical ordering:

$$c_G < c_A.$$

We can now determine the magnitude of the Compensating Differential required to eliminate the Gold Sector's Valorization Advantage.

For equality $g = \bar{g}$, we require:

$$\frac{N - \ell - d}{c_G + \ell + d} = \frac{N - \ell}{c_A + \ell}.$$

Solving for d gives:

$$\boxed{d^* = \frac{(N - \ell)(c_A - c_G)}{c_A + N}.}$$

The magnitude d^* is the **Required Compensating Differential**: the sector-specific additional value burden sufficient to equalize the two Rates of Valorization at the given realized state.

As long as positive Surplus Value $N - \ell > 0$ exists and $c_G < c_A$, we have $d^* > 0$.

The ordering of the two Rates of Valorization is therefore:

$$\begin{aligned} d < d^* &\implies g > \bar{g}, \\ d = d^* &\implies g = \bar{g}, \\ d > d^* &\implies g < \bar{g}. \end{aligned}$$

Structural Labour Disadvantage can therefore create a sector-specific barrier to Goldization.

Unlike an economy-wide increase in ℓ, the Compensating Differential acts only on the Gold Sector and can therefore directly alter its Rate of Valorization relative to that of the Non-Gold Economy.

22. Technical Cheapening of Compensation

The preceding chapter introduced the **Compensating Differential** d, the additional value borne by Gold Capital per normalized unit of labour in order to compensate for objectively worse labour conditions.

We now introduce Technical Progress into the production of the material means through which this compensation is provided.

Suppose that compensation for the Structural Labour Disadvantage requires an additional physical bundle of reproducible goods and infrastructure B_d.

This bundle may include additional housing, transport, protective equipment, infrastructure, and other reproducible material conditions required to compensate for the sector-specific labour disadvantage.

The classification used here is functional rather than purely physical.

A single material object may simultaneously perform a productive function and a labour-condition function.

Where this occurs, only the portion of its value attributable to compensating the Structural Labour Disadvantage enters d, while the portion required for production as such remains part of Constant Capital.

The same building, equipment, or infrastructure may therefore contain both components without eliminating their analytical distinction.

Let $P_d(t)$ denote the **Total Value of the Compensation Bundle** required per normalized unit of labour at economic state t.

By definition, $d(t) = P_d(t)$.

The physical Structural Labour Disadvantage may persist.

Accordingly, the additional physical bundle B_d need not disappear.

Technical Progress nevertheless raises productivity in the industries producing the reproducible elements of this bundle and reduces the total value required to reproduce it.

Within the idealized technical dynamics adopted in this work, the Total Value of the Compensation Bundle is reduced through realized finite economic changes:

$$P_d(t) \downarrow_E .$$

The economically relevant object is $P_d(t)$ itself, not the value of an arbitrarily selected individual component from which Economic Zero would then be transferred mechanically by multiplication.

Continued Technical Progress reduces the total value burden of the Compensation Bundle until that burden becomes negligible at its own relevant Objective Economic Scale:

$$P_d(t) \xrightarrow{\text{E}} 0_E.$$

Since:

$$d(t) = P_d(t),$$

the same magnitude expressed as the Compensating Differential satisfies:

$$\boxed{d(t) \xrightarrow{\text{E}} 0_E.}$$

This does not imply that the physical Structural Labour Disadvantage disappears.

Nor does it imply asymptotic approximation to arithmetic zero.

It means that the additional value expenditure required to compensate for the physical disadvantage is reduced through finite realized economic changes until that expenditure becomes negligible at the relevant Objective Economic Scale.

Thus the model distinguishes persistent physical disadvantage from the economically exhausted value cost of compensation.

The first may remain physically positive even when:

$$d(t) \xrightarrow{\text{E}} 0_E.$$

23. Failure of Structural Compensation

The preceding chapters introduced two distinct magnitudes.

The actual **Compensating Differential** $d(t)$ is the additional value burden actually borne by the Gold Sector in order to compensate for Structural Labour Disadvantage. The **Required Compensating Differential** $d^*(t)$ is the magnitude that would equalize the Gold and Non-Gold Rates of Valorization.

From Chapter 21:

$$d^*(t) = \frac{[N - \ell(t)][c_A(t) - c_G(t)]}{c_A(t) + N}.$$

As long as $N - \ell > 0$ and $c_G < c_A$, we have $d^* > 0$.

We first isolate the technical component of the movement by holding Competitive Total Labour Cost fixed, $\ell = \text{const}$.

For the Gold Sector, Chapter 9 established:

$$c_G(t) \downarrow_E .$$

Compensating Technical Deterioration does not reverse this value movement. Before the Minimum Viable Gold Technique becomes binding, $c_G \downarrow_E$ is accompanied by $\rho_G \downarrow_E$ in order to preserve the Current Reproduction Value of Gold. After the boundary becomes binding, $\rho_G = \underline{\rho}_G$, and further $c_G \downarrow_E$ instead produces Technical Devaluation, $p_G \downarrow_E$. In both cases, the value burden relevant to the present labour-cost comparison continues to move as $c_G(t) \downarrow_E$.

The effect of a change in Gold Constant Capital on the Required Compensating Differential is:

$$\frac{\partial d^*}{\partial c_G} = -\frac{N - \ell}{c_A + N} < 0.$$

Therefore:

$$c_G(t) \downarrow_E \Longrightarrow d^*(t) \uparrow_E$$

other things being equal.

The economic meaning is that a smaller Constant-Capital burden in the Gold Sector strengthens its technical Valorization Advantage.

A larger sector-specific Compensating Differential is therefore required to neutralize that advantage.

We now isolate the corresponding technical component in the Non-Gold Economy.

Part III retained the tendency toward technical deepening of Non-Gold Capital generated by Technical Competition.

For the normalized intersectoral comparison used here, and at fixed ℓ, we represent this technical deepening by $c_A(t) \uparrow_E$.

This is a local technical-direction statement at fixed Competitive Total Labour Cost.

It is not a claim that c_A must increase under every possible simultaneous historical change in all variables.

Its role is to represent the Constant-Capital component of Non-Gold technical deepening in the present comparative-static analysis.

The effect of this movement on the Required Compensating Differential is:

$$\frac{\partial d^*}{\partial c_A} = \frac{(N - \ell)(N + c_G)}{(c_A + N)^2} > 0.$$

Therefore:

$$c_A(t) \uparrow_E \Longrightarrow d^*(t) \uparrow_E$$

other things being equal.

Thus the technical movements $c_G(t) \downarrow_E$ and $c_A(t) \uparrow_E$ both increase the Required Compensating Differential at a fixed level of ℓ.

The actual Compensating Differential moves in the opposite direction.

Chapter 22 established:

$$d(t) \downarrow_E,$$

with:

$$d(t) \xrightarrow{\text{E}} 0_E.$$

We therefore obtain two opposing technical movements at fixed Competitive Total Labour Cost:

$$d(t) \downarrow_E,$$

while:

$$d^*(t) \uparrow_E .$$

Technical Progress simultaneously reduces the actual value burden of Structural Labour Disadvantage and raises the sector-specific value burden required to neutralize the Gold Sector's technical Valorization Advantage.

Suppose that at some realized state t_0, $d(t_0) = d^*(t_0)$. Then $g(t_0) = \bar{g}(t_0)$.

At the same fixed level of Competitive Total Labour Cost, a subsequent technical-development state reduces $d(t)$ while increasing the technical conditions represented by $d^*(t)$.

The previous equality is therefore not self-preserving.

Once $d(t) < d^*(t)$, we again obtain $g(t) > \bar{g}(t)$.

Once the actual Compensating Differential has been economically exhausted, $d(t) \xrightarrow{\text{E}} 0_E$, while $\ell < N$ and $c_G < c_A$, the Required Compensating Differential remains economically positive, $d^* > 0$.

Thus Structural Labour Disadvantage cannot permanently neutralize the Gold Sector's Valorization Advantage at a fixed economically positive level of Surplus Value.

This result is, however, one of **Comparative Statics**.

The complete Required Compensating Differential is:

$$d^*(t) = \frac{[N - \ell(t)][c_A(t) - c_G(t)]}{c_A(t) + N}.$$

Its full realized movement therefore depends simultaneously on $\ell(t)$, $c_G(t)$, and $c_A(t)$.

In particular, $\ell(t) \uparrow_E$ reduces $N - \ell(t)$ and can therefore reduce the Required Compensating Differential even while its technical component is strengthening.

The present chapter consequently does not assign an unconditional Economic Direction to the complete historical movement of $d^*(t)$. Instead, it establishes the technical movement that shifts the condition under which Structural Labour Disadvantage can neutralize the Gold Sector.

24. The Moving Labour-Compensation Threshold

Technical Progress changes the conditions under which Structural Labour Disadvantage can neutralize the Gold Sector's Valorization Advantage. The moving threshold also incorporates the movement of Competitive Total Labour Cost.

From Chapter 21:

$$d^*(t) = \frac{[N - \ell(t)][c_A(t) - c_G(t)]}{c_A(t) + N}.$$

Define the technical coefficient:

$$\phi(t) = \frac{c_A(t) - c_G(t)}{c_A(t) + N}.$$

Then:

$$d^*(t) = [N - \ell(t)]\phi(t).$$

On the post-crossing productive structure considered here, $c_G(t) < c_A(t)$, so $\phi(t) > 0$. Since $c_A - c_G < c_A + N$, we also obtain $0 < \phi(t) < 1$.

The magnitude $\phi(t)$ expresses the technical component of the Gold Sector's relative advantage arising from the difference in Constant-Capital burden between the normalized Gold and Non-Gold production units.

For the Gold Sector:

$$c_G(t) \downarrow_E .$$

Moreover:

$$\frac{\partial \phi}{\partial c_G} = -\frac{1}{c_A + N} < 0.$$

Therefore, other things being equal, $c_G(t) \downarrow_E$ raises $\phi(t)$.

For the Non-Gold Economy, the normalized technical-deepening component established in the preceding chapter gives, at fixed Competitive Total Labour Cost:

$$c_A(t) \uparrow_E .$$

Moreover:

$$\frac{\partial \phi}{\partial c_A} = \frac{N + c_G}{(c_A + N)^2} > 0.$$

Therefore, other things being equal, $c_A(t) \uparrow_E$ also raises $\phi(t)$.

The technical-development step therefore contains the joint Economic Direction:

$$c_G(t) \downarrow_E, \qquad c_A(t) \uparrow_E \Longrightarrow \phi(t) \uparrow_E .$$

This technical movement does not depend on the direction of Gold physical productivity itself. Before the Minimum Viable Gold Technique becomes binding, $\rho_G \downarrow_E$ compensates for $c_G \downarrow_E$ in the Current Reproduction Value of Gold. After that physical boundary becomes binding, $\rho_G = \underline{\rho}_G$, and further $c_G \downarrow_E$ produces $p_G \downarrow_E$. Neither case reverses the movement of c_G entering ϕ.

At the same time, Chapter 22 established:

$$d(t) \downarrow_E,$$

with:

$$d(t) \xrightarrow{\text{E}} 0_E.$$

The condition for equality between the Gold Rate of Valorization and the Non-Gold Rate of Valorization is:

$$d(t) = d^*(t).$$

Therefore:

$$d(t) = [N - \ell(t)]\phi(t).$$

Solving for ℓ, we obtain:

$$\boxed{\ell^*(t) = N - \frac{d(t)}{\phi(t)}.}$$

We call $\ell^*(t)$ the **Labour-Compensation Threshold**.

It is the level of Competitive Total Labour Cost required, for the given realized values of $d(t)$, $c_G(t)$, and $c_A(t)$, to neutralize the Gold Sector's Valorization Advantage exactly.

Three states follow directly.

The current sectoral ordering is:

$$\begin{aligned}
\ell(t) < \ell^*(t) &\Longrightarrow d(t) < d^*(t) \Longrightarrow g(t) > \bar{g}(t),\\
\ell(t) = \ell^*(t) &\Longrightarrow d(t) = d^*(t) \Longrightarrow g(t) = \bar{g}(t),\\
\ell(t) > \ell^*(t) &\Longrightarrow d(t) > d^*(t) \Longrightarrow g(t) < \bar{g}(t).
\end{aligned}$$

The first case preserves the Gold Sector's Valorization Advantage; the second exactly compensates it; in the third, Capital Selection ceases to direct capital toward the Gold Sector.

Now consider the movement of the threshold itself.

Suppose that at some realized state t_0, the two Rates of Valorization have been equalized, $\ell(t_0) = \ell^*(t_0)$. Over a subsequent realized technical-development step $t_1 > t_0$, Technical Progress produces $d(t_1) < d(t_0)$ and $\phi(t_1) > \phi(t_0)$.

Therefore:

$$\frac{d(t_1)}{\phi(t_1)} < \frac{d(t_0)}{\phi(t_0)}.$$

Consequently:

$$\ell^*(t_1) > \ell^*(t_0).$$

Technical Progress therefore shifts the Labour-Compensation Threshold upward.

A Competitive Total Labour Cost sufficient to equalize the two Rates at one realized state need not remain sufficient after further Technical Progress.

If $\ell(t_1) < \ell^*(t_1)$, then $g(t_1) > \bar{g}(t_1)$.

The Capital Selection Principle again directs capital toward the Gold Sector.

On this branch, $c_G < c_A$.

Renewed Goldization therefore generates positive net labour demand, as derived in Chapter 18, and produces $\ell(t) \uparrow_E$.

The system can therefore approach or cross the new threshold through realized finite labour-market adjustments. If a state satisfying $\ell_n < \ell^*_n$ is followed by $\ell_{n+1} \geq \ell^*_{n+1}$, the Labour-Compensation Threshold has undergone a **Finite Economic Crossing** $\ell \bowtie_E \ell^*$.

Exact attainment gives $g = \bar{g}$, while strict crossing gives $g < \bar{g}$. Thus a compensating adjustment is sufficient to neutralize or reverse the Gold Sector's Valorization Advantage whenever $\ell \geq \ell^*$.

Under strict crossing:

$$g < \bar{g} \xrightarrow{\text{E}} \text{Capital Motion toward the Non-Gold Economy.}$$

Continued Technical Progress can shift $\ell^*(t)$ above a previously sufficient Total Labour Cost. If $\ell < \ell^*$ re-emerges, then $g > \bar{g}$ and Goldization resumes. The relevant distinction is whether labour-market adjustment attains or crosses the moving threshold.

The remaining Labour-Compensation Threshold gap is:

$$N - \ell^*(t) = \frac{d(t)}{\phi(t)}.$$

The established movements of $d(t)$ and $\phi(t)$ imply, between successive realized technical-development states:

$$d_{n+1} < d_n, \qquad 0 < \phi_n < \phi_{n+1}.$$

Therefore:

$$0 < \frac{d_{n+1}}{\phi_{n+1}} < \frac{d_n}{\phi_n}.$$

Hence:

$$N - \ell^*(t) = \frac{d(t)}{\phi(t)} \downarrow_E .$$

The model introduces no positive lower floor or autonomous attenuation mechanism for this gap. Under the finite-boundary principles established in Chapter 0, its continued realized reduction therefore gives:

$$\boxed{N - \ell^*(t) = \frac{d(t)}{\phi(t)} \xrightarrow{\mathrm{E}} [0_E]_\mathrm{T}.}$$

This is a Terminal Economic Boundary of the threshold gap itself, not a mechanical transfer of Economic Zero from $d(t)$, and it does not require asymptotic approximation of $\ell^*(t)$ to N.

Exact attainment or finite overshoot may occur at particular realized states, but neither constitutes a stable positive terminal state. The terminal implications therefore depend on whether labour-market adjustment fails or repeatedly succeeds in attaining or crossing the moving threshold.

If labour-market adjustment fails to attain or cross the moving threshold, $\ell(t) < \ell^*(t)$, then $g(t) > \bar{g}(t)$ and Goldization continues.

If labour-market adjustment repeatedly remains strong enough to attain or cross the moving threshold, then every successful compensating state satisfies $\ell(t) \geq \ell^*(t)$, and therefore:

$$N - \ell(t) \leq N - \ell^*(t).$$

But:

$$N - \ell^*(t) \xrightarrow{\mathrm{E}} [0_E]_\mathrm{T}.$$

Hence repeated successful compensation cannot preserve economically positive Surplus Value beyond the Terminal Economic Boundary of the threshold gap.

At or before the state at which:

$$N - \ell^*(t) \xrightarrow{\mathrm{E}} [0_E]_\mathrm{T},$$

we obtain:

$$N - \ell(t) \xrightarrow{\mathrm{E}} [0_E]_\mathrm{T},$$

or a finite labour-market adjustment crosses the positive-Surplus-Value boundary directly.

For the Non-Gold Economy:

$$\bar{g}(t) = \frac{N - \ell(t)}{c_A(t) + \ell(t)}.$$

Therefore positive Non-Gold Valorization reaches its Terminal Economic Boundary:

$$\bar{g}(t) \xrightarrow{\mathrm{E}} [0_E]_\mathrm{T}.$$

Moreover, every successful compensating state satisfies $\ell(t) \geq \ell^*(t)$ and therefore $g(t) \leq \bar{g}(t)$.

Consequently, once the Non-Gold Rate of Valorization reaches its Terminal Economic Boundary, the Gold Rate of Valorization cannot remain economically positive:

$$g(t) \xrightarrow{\mathrm{E}} [0_E]_\mathrm{T}.$$

Hence:

$$\boxed{g(t), \bar{g}(t) \xrightarrow{\mathrm{E}} [0_E]_{\mathrm{T}}.}$$

Structural Labour Disadvantage therefore does not create a stable positive resolution of the Refrigerator Problem.

Part VI.
The Reproductive Boundary

25. Material Reproductive Requirement

Until now, the expansion of the Gold Sector has been analyzed through the movement of capital and labour between sectors.

We now introduce the minimum volume of material production without which the reproduction of the economy cannot continue.

Let D denote the **Required Material Output**: the minimum quantity of material goods that must be produced by the Non-Gold Economy over the comparable period in order to sustain social reproduction. Assume $0 < D < AL$, so the potential material output of the Non-Gold Economy under full use of the available normalized social productive resource exceeds the Required Material Output.

The magnitude D includes the material product required for the reproduction of labour power and for the continuation of the material conditions of production.

Now let $x(t)$ denote the share of the available normalized social productive resource allocated to the Gold Sector through the movement of social capital. The corresponding share remaining in the Non-Gold Economy is $1 - x(t)$. Let L denote the total available normalized social productive resource distributed between the two sectors. The Gold Sector therefore uses $x(t)L$, while the Non-Gold Economy uses $[1 - x(t)]L$.

At this stage, the realized material productivity of the Non-Gold Economy is held fixed at A, the quantity of material output produced by one unit of normalized productive resource over the comparable period. Thus $A = \text{const}$.

Material output of the Non-Gold Economy is therefore:

$$Y_A(t) = A[1 - x(t)]L.$$

For social reproduction to be maintained, it is necessary that:

$$Y_A(t) \geq D.$$

Equivalently:

$$A[1 - x(t)]L \geq D.$$

The material-reproduction condition depends on the Required Material Output D, the available normalized social productive resource L, the Non-Gold material productivity A, and the allocation share $x(t)$. Gold physical productivity ρ_G does not enter this identity directly. The corresponding maximum Gold-sector allocation compatible with $Y_A(t) \geq D$ is derived next.

26. Reproductive Boundary

From the necessary material-reproduction condition:

$$A[1 - x(t)]L \geq D,$$

we can determine the maximum share of the normalized social productive resource that may be allocated to the Gold Sector while the Required Material Output is maintained.

Since $A > 0$ and $L > 0$, division by AL gives:

$$1 - x(t) \geq \frac{D}{AL}.$$

Therefore:

$$x(t) \leq 1 - \frac{D}{AL}.$$

Define:

$$\boxed{x_R = 1 - \frac{D}{AL}.}$$

From Chapter 25, $0 < D < AL$. Hence $0 < \frac{D}{AL} < 1$, and therefore $0 < x_R < 1$.

The magnitude x_R is called the **Reproductive Boundary**.

It represents the maximum share of the normalized social productive resource that may be allocated to the Gold Sector while the Non-Gold Economy still produces at least the Required Material Output at the given productivity A.

The Reproductive Boundary partitions the material-reproduction states as:

$$\boxed{\begin{aligned} x < x_R &\iff Y_A > D, \\ x = x_R &\iff Y_A = D, \\ x > x_R &\iff Y_A < D. \end{aligned}}$$

The first region contains a positive Reproductive Margin. At $x = x_R$, the Required Material Output is maintained exactly; Reproductive Breakdown occurs only after the boundary is crossed into $x > x_R$.

Because Goldization proceeds through realized finite economic changes, the trajectory is not required to occupy $x = x_R$ as a separate realized state.

It may move directly from $x_n < x_R$ to $x_{n+1} > x_R$.

The Reproductive Boundary is therefore the **Transit Economic Boundary** $[x_R]_C$.

Crossing this boundary does not terminate the variable x. It moves the economy from the reproductively admissible region $x \leq x_R$ into $x > x_R$, where $Y_A < D$.

Thus the finite Reproductive Boundary separates exact material reproduction from actual Reproductive Breakdown.

For fixed D, A, and L, the boundary itself is fixed, $x_R = \text{const}$.

27. Goldization under Fixed Non-Gold Productivity

Throughout this section $A = \text{const}$, so the previously derived Reproductive Boundary $x_R = 1 - \frac{D}{AL}$ is fixed. Gold-sector technical dynamics, including changes in ρ_G, do not enter this boundary directly.

Now consider a realized state on which $g > \bar{g}$.

The Capital Selection Principle gives:

$$g > \bar{g} \xrightarrow{\mathrm{E}} \text{Capital Motion toward the Gold Sector.}$$

As capital moves toward Gold, the share of the normalized social productive resource allocated to the Gold Sector rises through successive finite reallocations, $x(t) \uparrow_E$.

We call this process **Goldization**: the successive reallocation of social capital and productive resources from the Non-Gold Economy into the Gold Sector.

Goldization therefore produces $1 - x(t) \downarrow_E$. At fixed A, material output of the Non-Gold Economy correspondingly moves as:

$$Y_A(t) = A[1 - x(t)]L \downarrow_E .$$

While $g > \bar{g}$ continues to direct capital toward the Gold Sector, Goldization moves the allocation share toward the fixed Reproductive Boundary:

$$\boxed{x(t) \xrightarrow{\mathrm{E}} [x_R]_{\mathrm{C}}.}$$

The boundary cases have already been established: exact attainment $x = x_R$ gives $Y_A = D$, while crossing into $x > x_R$ gives $Y_A < D$ and constitutes Reproductive Breakdown. If labour-market compensation instead produces $g \leq \bar{g}$, Goldization may pause or reverse before the boundary is crossed.

The next Part removes the fixed-productivity assumption for the Non-Gold Economy and determines how Realized Technical Development moves the Reproductive Boundary itself.

Part VII. The Moving Reproductive Boundary

28. Moving Reproductive Boundary

The preceding Part treated the material productivity of the Non-Gold Economy as fixed $A = \text{const}$. We now remove this assumption.

Let $A = A(t)$ denote the **Realized Material Productivity of the Non-Gold Economy**: the quantity of material output actually produced by one unit of normalized productive resource under techniques that have been materially incorporated into Non-Gold production.

Here $A(t)$ denotes realized material productivity in the Non-Gold Economy, whereas $\rho_G(t)$ denotes the physical productivity of Gold production. The Reproductive Boundary depends directly on $A(t)$, because the Non-Gold Economy produces the material output required for social reproduction; changes in $\rho_G(t)$ do not enter that boundary directly.

A technical possibility, invention, or newly discovered productive method does not by itself raise $A(t)$.

Productivity rises only when a more productive Non-Gold technique has actually been implemented, reproduced, and diffused through the productive structure.

We therefore represent realized Non-Gold Technical Progress economically by:

$$A(t) \uparrow_E .$$

In continuous analytical notation, the same Economic Direction may be represented by:

$$A'(t) > 0.$$

This derivative represents the direction already established economically.

It does not define the underlying movement as infinitesimal.

Over successive realized economic states, the same unit of normalized productive resource in the Non-Gold Economy becomes capable of producing a larger quantity of material output.

The condition of material reproduction is now:

$$A(t)[1 - x(t)]L \geq D.$$

Hence:

$$1 - x(t) \geq \frac{D}{A(t)L}.$$

The Reproductive Boundary therefore becomes:

$$\boxed{x_R(t) = 1 - \frac{D}{A(t)L}.}$$

For fixed D and L, rising Realized Material Productivity $A(t) \uparrow_E$ moves the Reproductive Boundary outward, $x_R(t) \uparrow_E$.

The same relation may be represented continuously by:

$$\frac{dx_R}{dt} = \frac{D}{LA(t)^2} A'(t).$$

Since:

$$A'(t) > 0,$$

the continuous representation gives:

$$\frac{dx_R}{dt} > 0.$$

Again, the derivative represents analytically an Economic Direction already established through realized finite changes.

A more productive Non-Gold Economy can produce the same Required Material Output D with a smaller share of the total normalized social productive resource.

The minimum share that must remain in the Non-Gold Economy is:

$$1 - x_R(t) = \frac{D}{A(t)L}.$$

As $A(t) \uparrow_E$, we obtain $1 - x_R(t) \downarrow_E$.

Realized Technical Progress in the Non-Gold Economy therefore shifts outward the Reproductive Boundary at which further Goldization begins to disrupt necessary material reproduction.

The outward movement of $x_R(t)$ occurs through increases in $A(t)$ that have been materially realized and incorporated into Non-Gold production.

Gold-sector technical development remains distinct from this movement. Before the Minimum Viable Gold Technique becomes binding, $\rho_G(t)$ may fall through Compensating Technical Deterioration; after that boundary is reached, further cheapening of c_G may instead reduce p_G. Neither regime enters the Reproductive Boundary directly:

$$x_R(t) = 1 - \frac{D}{A(t)L}.$$

The relation relevant for the subsequent analysis is therefore between $x(t)$, the actual share of normalized social productive resources allocated to the Gold Sector, and $x_R(t)$, the Reproductive Boundary generated by the realized material productivity of the Non-Gold Economy.

29. Capital Motion and Realized Technical Development

Realized Non-Gold Technical Development moves the Reproductive Boundary outward:

$$A(t) \uparrow_E \xrightarrow{\text{E}} x_R(t) \uparrow_E .$$

The increase in $A(t)$ requires material implementation through the movement of capital.

A technique does not raise $A(t)$ merely because it has been invented, discovered, or made technically possible.

It raises realized material productivity only after capital has materially implemented and reproduced it within the productive structure.

Such implementation requires actual movement of capital.

New means of production must be produced and purchased, existing productive capacities must be replaced or re-equipped, new productive complexes may have to be constructed, and more productive techniques must be materially diffused through production.

Let $J_K(t)$ denote the **Speed of Total Capital Motion**: the rate at which the total realized movement of social capital proceeds through reproduction, replacement, expansion, reorganization, technical implementation, and sectoral reallocation over the common comparison period.

Within this Total Capital Motion, let $J_D(t)$ denote the **Speed of Realized Technical Development**: the speed of the component of Total Capital Motion through which productivity-raising techniques are materially implemented and diffused in the Non-Gold productive structure so as to raise $A(t)$.

Thus $J_D(t)$ measures a proper component of $J_K(t)$: every realized act through which a productivity-raising Non-Gold technique is materially implemented and diffused is itself an act of Total Capital Motion.

Let $J_O(t)$ denote the speed of all remaining forms of realized Capital Motion.

The speed of Total Capital Motion therefore decomposes as:

$$\boxed{J_K(t) = J_D(t) + J_O(t).}$$

Here $J_O(t)$ contains realized movements that do not constitute the productivity-raising Non-Gold Technical Development represented by $J_D(t)$.

These include continued reproduction of existing production, ordinary replacement without productivity-raising technical transformation, expansion, organizational reconfiguration, sectoral reallocation, and other realized technical adjustments that do not themselves raise $A(t)$.

In particular, Compensating Technical Deterioration in the Gold Sector is not counted as J_D.

It is a realized technical reconfiguration of Gold production, but it does not constitute the productivity-raising Non-Gold Technical Development whose realized effect is:

$$A(t) \uparrow_E .$$

It therefore remains within Total Capital Motion without being part of the J_D component defined here.

In a reproducing capitalist economy, the remaining forms of Capital Motion are economically positive:

$$J_O(t) > 0.$$

Hence:

$$J_K(t) > J_D(t).$$

This strict relation constitutes the **Capital-Development Principle**:

$$\boxed{\text{Realized Technical Development} \subsetneq \text{Total Capital Motion.}}$$

The distinction between **Potential Innovation** and **Realized Technical Development** is therefore essential.

A technique may exist as knowledge or as a technically available possibility without yet altering the productive structure.

In that case it does not raise $A(t)$. Only when capital materially implements and diffuses the technique does it enter realized Non-Gold production and affect material productivity.

Consequently:

$$J_D(t) > 0$$

is a necessary economic condition for:

$$A(t) \uparrow_E .$$

In continuous analytical notation, the resulting realized productivity growth may be represented by:

$$A'(t) > 0.$$

The outward Economic Direction of the Reproductive Boundary is generated through this realized component of Capital Motion. When:

$$J_D(t) > 0,$$

its completed material implementation produces:

$$J_D(t) > 0 \xrightarrow{\mathrm{E}} A(t) \uparrow_E \xrightarrow{\mathrm{E}} x_R(t) \uparrow_E .$$

Thus the outward movement of $x_R(t)$ is generated through Realized Technical Development already contained within Total Capital Motion.

Of particular importance is **Replacement Investment**.

Productivity-raising techniques enter realized production through replacement and re-equipment of existing capital and through new investment.

The same Total Capital Motion also reproduces existing production, expands productive capacity, reorganizes production, and reallocates capital among sectors.

Technical Development and sectoral reallocation therefore occur within one Total Capital Motion, even though they represent different components of it.

The **speed of Capital Motion** is distinct from the **sectoral direction** in which that motion is realized.

The magnitude $J_K(t)$ does not measure a change in the Gold share as such. It measures the speed at which social capital is materially reproduced, replaced, reorganized, expanded, re-equipped, and reallocated through the productive structure over the common comparison period. This motion exists even when the sectoral composition of the economy remains unchanged.

Consider, for example, the replacement of an existing productive complex whose productive life has been exhausted. Capital may reproduce the same type of Non-Gold productive complex:

$$\text{Non-Gold Capital} \longrightarrow \text{Non-Gold Capital},$$

or it may rematerialize the same released productive resources as a Gold-producing complex:

$$\text{Non-Gold Capital} \longrightarrow \text{Gold Capital}.$$

These are not two different kinds of capital mobility. Both are realized acts of Capital Motion. The difference lies in the *direction of rematerialization*. In the first case, Capital Motion reproduces the existing sectoral structure and therefore need not change $x(t)$. In the second case, the same underlying capacity of capital to move and rematerialize alters the sectoral structure and raises $x(t)$. The speed of Capital Motion is therefore distinct from its sectoral direction.

A positive $J_K(t)$ may therefore coexist with $\dot{x}(t) = 0$ when Capital Motion reproduces the existing sectoral allocation. Only after an economic mechanism assigns a systematic sectoral direction to Total Capital Motion does that same speed acquire a corresponding representation through the movement of $x(t)$.

Realized Technical Development remains the productivity-raising component of Total Capital Motion: $J_D(t)$ measures the component whose completed material realization raises $A(t)$, whereas $J_K(t)$ measures Total Capital Motion as a whole. Capital Motion may therefore occur without an increase in Non-Gold material productivity.

The sectoral direction of Total Capital Motion depends on the relative Rates of Valorization of the Gold Sector and the Non-Gold Economy.

30. Dynamic Capital Reallocation

Recall that $x(t)$ denotes the share of normalized social productive resources allocated to the Gold Sector.

Define the **Valorization Differential**:

$$\Delta(t) = g(t) - \bar{g}(t).$$

After the introduction of Structural Labour Disadvantage, the continuing Goldization branch is characterized by $d(t) < d^*(t)$. Under this condition, $\Delta(t) > 0$ and therefore $g(t) > \bar{g}(t)$.

The Gold Sector then possesses the higher Rate of Valorization.

Under the **Capital Selection Principle**, this positive differential gives Total Capital Motion a definite sectoral direction:

$$\Delta(t) > 0 \xrightarrow{\text{E}} \text{Total Capital Motion toward the Gold Sector.}$$

Consequently, $x(t) \uparrow_E$.

This realized finite reallocation of social productive resources toward the Gold Sector is **Goldization**.

Conversely, if $\Delta(t) < 0$, then:

$$\Delta(t) < 0 \xrightarrow{\text{E}} \text{Total Capital Motion toward the Non-Gold Economy,}$$

and therefore $x(t) \downarrow_E$.

If $\Delta(t) = 0$, neither sector possesses a Valorization Advantage, and no directional Economic Transition follows from the Capital Selection Principle alone.

The Refrigerator mechanism analyzed below concerns the continuing branch on which $\Delta(t) > 0$.

The Capital-Development Principle gives $J_K(t) = J_D(t) + J_O(t)$ with $J_O(t) > 0$, and therefore $J_K(t) > J_D(t)$.

Here the J-notation expresses the speeds of the economic processes already established.

The equalities introduced below are **branch-specific speed representations**, not general definitions of Capital Motion.

In particular, $J_K(t) = \dot{x}(t)$ does not mean that Capital Motion exists only when the Gold share changes. As established in the preceding chapter, Total Capital Motion may remain positive while reproducing an unchanged sectoral structure, in which case $J_K(t) > 0$ while $\dot{x}(t) = 0$.

The equality becomes applicable only on the continuing Goldization branch because the Capital Selection Principle assigns a definite Goldward direction to Total Capital Motion. Once $\Delta(t) > 0$ directs the realized rematerialization of capital toward the Gold Sector, the sectoral expression of that directed Total Capital Motion is the increase in $x(t)$.

At the common Objective Economic Scale used for the reproductive comparison, its branch-specific speed representation is therefore:

$$J_K(t) = \dot{x}(t).$$

The same principle applies to Realized Technical Development. The magnitude $J_D(t)$ is not an independent rate of invention, knowledge accumulation, or potential technical possibility. It is the speed of the realized component of Capital Motion through which the Non-Gold productive structure is materially transformed so as to raise $A(t)$. The reproductive effect of that realized transformation is the outward displacement of $x_R(t)$.

Accordingly, on the same normalized Objective Economic Scale, its branch-specific reproductive speed representation is:

$$J_D(t) = \dot{x}_R(t).$$

These representations therefore compare two realized transformations of the same productive structure on a common economic scale:

$$J_K = \text{Total Capital Motion directed toward Goldization,}$$

$$J_D = \text{the productivity-raising component of that Capital Motion.}$$

Since:

$$J_K(t) = J_D(t) + J_O(t), \qquad J_O(t) > 0,$$

Realized Technical Development cannot exhaust Total Capital Motion:

$$J_D(t) < J_K(t).$$

Thus, once Total Capital Motion is directionally expressed as Goldization while Realized Technical Development is expressed through the movement of the Reproductive Boundary, the same proper-component relation becomes:

$$\boxed{\dot{x}_R(t) < \dot{x}(t).}$$

The inequality follows because Goldization expresses the directed motion of Total Capital, whereas movement of the Reproductive Boundary requires the narrower productivity-raising transformation realized within that same Capital Motion.

Realized Technical Development raises $A(t)$ and moves the Reproductive Boundary outward, while directed Total Capital Motion raises the Gold-sector allocation share:

$$x(t) \uparrow_E, \qquad x_R(t) \uparrow_E .$$

Because $J_D(t)$ remains a proper component of $J_K(t)$ on this branch, $x(t)$ advances faster than $x_R(t)$ at the common Objective Economic Scale. There is therefore no subsequent Speed Crossing between Goldization and Realized Technical Development on the continuing branch.

The Gold-sector technical phase does not alter this reproductive ordering by itself.

Before $T_{\min}$, Gold production may exhibit $c_G \downarrow_E$ and $\rho_G \downarrow_E$, while Compensating Technical Deterioration preserves the Current Reproduction Value of Gold.

After $T_{\min}$, further $c_G \downarrow_E$ may instead produce $p_G \downarrow_E$.

Neither regime enters $x_R(t) = 1 - \frac{D}{A(t)L}$ directly.

The Reproductive Boundary continues to be determined by:

$$x_R(t) = 1 - \frac{D}{A(t)L}.$$

Thus, as long as Idealized Gold remains operative and $\Delta(t) > 0$, the reproductive race remains between $x(t)$ and $x_R(t)$, regardless of whether the Gold Sector is still on its compensating technical branch or has already reached the Minimum Viable Gold Technique.

Technical Progress moves the Reproductive Boundary outward, but it does not constitute an independent economic process capable of moving that boundary outward faster than the Total Capital Motion within which Realized Technical Development itself occurs.

Part VIII. The Refrigerator Mechanism

31. General Refrigerator Mechanism

By this point, the principal components of the **Refrigerator Problem** have been derived separately. They can now be combined into a single dynamic structure.

Idealized Gold combines Direct Monetary Realization with a Current Reproduction Value. Under unchanged current value conditions, Gold production initially exhibits the Gold-Productivity Status Quo:

$$\rho_G = \rho_G^0.$$

General Technical Progress subsequently gives:

$$c_G(t) \downarrow_E, \qquad v^0(t) \downarrow_E, \qquad g^0(t) \uparrow_E .$$

Before the Minimum Viable Gold Technique becomes binding, cheapening of Gold Constant Capital is accompanied by Compensating Technical Deterioration:

$$c_G(t) \downarrow_E \xrightarrow{\mathrm{E}} \rho_G(t) \downarrow_E, \qquad p_G = \text{const}.$$

This compensating branch reaches:

$$\rho_G \xrightarrow{\mathrm{E}} [\underline{\rho}_G]_{\mathrm{T}}.$$

Let $T_{\min}$ denote the first realized state at which this physical boundary becomes binding. Beyond $T_{\min}$, further cheapening of c_G can no longer be offset by lower Gold productivity and instead produces the Technical Devaluation Regime:

$$p_G \downarrow_E, \qquad g^0 \uparrow_E .$$

On the standing branch in which Idealized Gold remains economically operative, the Profit-Rate Crossing gives:

$$g^0 \bowtie_E \bar{g}.$$

On the post-crossing branch:

$$g^0 > \bar{g} \iff c_G < c_A.$$

After Labour Competition and Structural Labour Disadvantage are incorporated, the current Valorization Differential is:

$$\Delta(t) = g(t) - \bar{g}(t).$$

On the continuing Goldization branch, $\Delta(t) > 0$, and the Capital Selection Principle gives:

$$\Delta(t) > 0 \xrightarrow{\text{E}} \text{Total Capital Motion toward the Gold Sector} \xrightarrow{\text{E}} x(t) \uparrow_E .$$

The rise in $x(t)$ is Goldization.

Structural Labour Disadvantage is summarized by the Compensating Differential $d(t)$ and the equalizing value:

$$d^*(t) = \frac{[N - \ell(t)][c_A(t) - c_G(t)]}{c_A(t) + N}.$$

With:

$$\phi(t) = \frac{c_A(t) - c_G(t)}{c_A(t) + N},$$

the corresponding Labour-Compensation Threshold is:

$$\ell^*(t) = N - \frac{d(t)}{\phi(t)}.$$

The threshold partitions the current sectoral ordering:

$$\ell < \ell^* \Rightarrow g > \bar{g}, \qquad \ell = \ell^* \Rightarrow g = \bar{g}, \qquad \ell > \ell^* \Rightarrow g < \bar{g}.$$

Because realized labour-market adjustments are finite, the threshold may be attained or crossed. A compensating state that neutralizes or reverses the Gold Sector's Valorization Advantage therefore satisfies:

$$\ell(t) \geq \ell^*(t).$$

The preceding analysis established:

$$d(t) \xrightarrow{\text{E}} 0_E, \qquad N - \ell^*(t) = \frac{d(t)}{\phi(t)} \xrightarrow{\text{E}} [0_E]_{\text{T}}.$$

Whenever $\ell < \ell^*$, the Gold Sector retains its Valorization Advantage and Goldization resumes. The positive net labour demand generated by Goldization, derived in Chapter 18, raises Competitive Total Labour Cost:

$$\ell(t) \uparrow_E .$$

If labour-market compensation repeatedly attains or crosses the moving threshold, then:

$$\ell(t) \geq \ell^*(t), \qquad N - \ell(t) \leq N - \ell^*(t).$$

Hence repeated successful compensation gives:

$$N - \ell(t) \xrightarrow{\text{E}} [0_E]_{\text{T}},$$

or crosses the positive-Surplus-Value boundary directly. The corresponding sectoral Rates of Valorization therefore reach:

$$g(t), \bar{g}(t) \xrightarrow{\mathrm{E}} [0_E]_{\mathrm{T}}.$$

This is the **Valorization Boundary**. Let T_V denote the first realized economic state at which positive capitalist valorization reaches this Terminal Economic Boundary.

The material side of the mechanism is summarized by:

$$Y_A(t) = A(t)[1 - x(t)]L, \qquad x_R(t) = 1 - \frac{D}{A(t)L}.$$

Realized Technical Development moves $x_R(t)$ outward, while the Capital-Development Principle on the continuing Goldization branch gives:

$$J_K(t) > J_D(t), \qquad \dot{x}(t) > \dot{x}_R(t).$$

Define the **Reproductive Margin**:

$$M_R(t) = x_R(t) - x(t).$$

The relative movement of $x(t)$ and $x_R(t)$ therefore gives:

$$M_R(t) \downarrow_E .$$

Its material interpretation is:

$$M_R > 0 \iff Y_A > D, \qquad M_R = 0 \iff Y_A = D, \qquad M_R < 0 \iff Y_A < D.$$

The Reproductive Boundary is the Transit Economic Boundary:

$$M_R \xrightarrow{\mathrm{E}} [0]_{\mathrm{C}}.$$

Exact attainment $M_R = 0$ still reproduces the Required Material Output. **Reproductive Breakdown** begins only after a realized finite crossing into $M_R < 0$.

Let T_R denote the first realized economic state at which:

$$M_R(T_R) < 0, \qquad x(T_R) > x_R(T_R), \qquad Y_A(T_R) < D.$$

The continuing Idealized-Gold trajectory therefore contains two systemic boundary movements:

$$g, \bar{g} \xrightarrow{\mathrm{E}} [0_E]_{\mathrm{T}}, \qquad M_R \xrightarrow{\mathrm{E}} [0]_{\mathrm{C}}.$$

The first defines the Valorization Boundary T_V; crossing the second produces Reproductive Breakdown at T_R.

The state $T_{\min}$ has a different role: it is the phase boundary at which Compensating Technical Deterioration is exhausted, not a third capitalist terminal boundary. If $T_R < T_{\min}$ or $T_V < T_{\min}$, the corresponding systemic boundary is realized before the Minimum Viable Gold Technique becomes binding. If $T_{\min} < T_R$ and $T_{\min} < T_V$, the Technical Devaluation Regime begins first while the Refrigerator dynamics continue on the standing branch.

32. Reproductive Breakdown

Consider first the branch on which Idealized Gold remains economically operative and Reproductive Breakdown occurs before the Valorization Boundary, $T_R < T_V$.

At T_R, the Reproductive Boundary has been crossed:

$$M_R(T_R) < 0, \qquad x(T_R) > x_R(T_R), \qquad Y_A(T_R) < D.$$

An exact boundary state $M_R = 0$, equivalently $x = x_R$ and $Y_A = D$, still maintains the Required Material Output and is not Reproductive Breakdown. Because $T_R < T_V$, positive capitalist valorization has not yet reached its Terminal Economic Boundary, although the finite transition into T_R may also alter the current ordering of g and $\bar{g}$.

Define the **Reproductive Deficit**:

$$R(t) = D - Y_A(t).$$

At exact boundary attainment $R = 0$; after crossing $R > 0$.

The Non-Gold Economy then fails to produce the minimum material product required for the reproduction of labour power and the continuation of the material conditions of production.

Gold production may still possess positive Surplus Value and a positive Rate of Valorization at this state.

But a monetary product cannot replace the missing material product, $Y_A < D$.

The system has therefore crossed the finite material boundary required for its own reproduction.

This is **Reproductive Breakdown**.

If Idealized Gold had been abandoned before T_R, the Idealized-Gold model would instead have terminated by abandonment and this particular Reproductive Breakdown would not have been realized within that monetary regime.

33. Extinction of Valorization

Consider now the branch on which Idealized Gold remains economically operative and the Valorization Boundary is reached before Reproductive Breakdown, $T_V < T_R$.

The moving Labour-Compensation Threshold already established that repeated successful compensation carries positive Surplus Value to its Terminal Economic Boundary:

$$N - \ell(t) \xrightarrow{\mathrm{E}} [0_E]_{\mathrm{T}},$$

or across that boundary through a finite adjustment. Consequently:

$$g(t), \bar{g}(t) \xrightarrow{\mathrm{E}} [0_E]_{\mathrm{T}}.$$

This is the **Extinction of Valorization**.

Economic Zero does not require $g = \bar{g} = 0$ as an exact arithmetic identity.

It means that positive capitalist valorization has become economically negligible at the relevant Objective Economic Scale and therefore no longer sustains the capitalist self-expansion represented by the model.

Because $T_V < T_R$, Reproductive Breakdown has not yet occurred:

$$M_R(T_V) \geq 0 \iff x(T_V) \leq x_R(T_V) \iff Y_A(T_V) \geq D.$$

Material reproduction may therefore remain above the Required Material Output or stand exactly at the Reproductive Boundary when positive capitalist valorization reaches its Terminal Economic Boundary.

If Idealized Gold had been abandoned before T_V, the Idealized-Gold model would instead have terminated by abandonment and the Extinction of Valorization derived here would not have been realized within that monetary regime.

34. Refrigerator Dilemma under Continued Idealized Gold

We can now state the Refrigerator Dilemma precisely.

The Dilemma concerns the branch on which Idealized Gold continues to function as money.

On that branch, two systemic terminal events have been defined.

The **Valorization Boundary** occurs at T_V, while the first realized state of **Reproductive Breakdown** occurs at T_R.

The possible orderings are exhaustive:

$$\boxed{T_V < T_R, \qquad T_R < T_V, \qquad \text{or} \qquad T_V = T_R.}$$

If $T_V < T_R$, Extinction of Valorization occurs while $Y_A(T_V) \geq D$. If $T_R < T_V$, Reproductive Breakdown occurs while positive capitalist valorization has not yet reached its Terminal Economic Boundary. If $T_V = T_R$, both terminal events are realized at the same economic state:

$$g, \bar{g} \xrightarrow{\mathrm{E}} [0_E]_{\mathrm{T}}, \qquad M_R(T_R) < 0, \qquad Y_A(T_R) < D.$$

The preceding analysis also rules out Realized Technical Development and Structural Labour Disadvantage as stable positive terminal escapes. Realized Technical Development remains a proper component of Total Capital Motion, while labour compensation either fails to neutralize the Gold Valorization Advantage or repeatedly reaches the moving threshold and carries positive Surplus Value to its Terminal Economic Boundary.

The state $T_{\min}$ is a technical phase boundary rather than an additional Refrigerator terminal event. If it occurs before T_V and T_R, the Technical Devaluation Regime begins; possible abandonment of Idealized Gold then belongs to the termination of the monetary configuration analyzed in the next section.

35. End of the Idealized-Gold Model

The object analyzed in this work is not capitalism in abstraction from every monetary form.

It is the specific joint configuration of Capitalism with Idealized Gold.

The **End of the Model** therefore means that this joint configuration ceases to exist.

It does not necessarily mean that every possible capitalist economy ceases to exist.

There are three general modes through which the Idealized-Gold model can end.

The first is **Economic Abandonment of Idealized Gold**.

If $T_{\min}$ is reached while the system still operates with Idealized Gold, further technical cheapening of Gold Constant Capital produces $p_G \downarrow_E$.

Because Idealized Gold possesses Monetary Persistence, recurrent reductions in its Current Reproduction Value reduce the value represented by each economically identical unit of Idealized Gold.

Gold may nevertheless continue to function as money for some realized interval.

The state $T_{\min}$ does not itself imply immediate monetary abandonment.

However, if recurrent and expected Technical Devaluation becomes incompatible with the continued monetary use of Gold, economic agents cease to treat Idealized Gold as the universal equivalent.

Economic abandonment is the general cessation of the monetary function of Idealized Gold.

In that case, Idealized Gold ceases to function as money.

Its special Gold-Sector role in the present model thereby disappears.

Gold production need not first become unprofitable.

Indeed, the Gold Rate of Valorization may remain positive or continue to rise while the money commodity itself becomes increasingly unsuitable as monetary wealth.

Economic abandonment may also occur anticipatorily if the economic system ceases to retain Idealized Gold after the terminal consequences of continued Goldization become economically apparent.

The present model does not impose a unique behavioral timing for such anticipatory abandonment.

It identifies abandonment as a possible termination of the Idealized-Gold configuration.

The second mode is **Institutional Abandonment of Idealized Gold**.

A state, monetary authority, or other institutional structure may replace Idealized Gold with another monetary form before Economic Abandonment, Reproductive Breakdown, or Extinction of Valorization is realized.

Institutional Abandonment enters the model as an external exit from the monetary regime being analyzed. The Refrigerator mechanism does not endogenously require it.

Once Idealized Gold is no longer the money commodity, Capitalism with Idealized Gold no longer exists.

Capitalism may continue under another monetary form, but the present model has ended.

The third mode is **Capitalist Terminal Failure under Continued Idealized Gold**. If the money commodity is not abandoned, the Refrigerator Dilemma applies: the system reaches T_V, T_R, or both at the same realized state. The first marks the Terminal Economic Boundary of positive capitalist valorization; the second records Reproductive Breakdown $Y_A < D$. These events terminate the capitalist process represented within the model rather than merely replacing its monetary form.

To express the common termination structure, let T_A denote the first realized economic state at which Idealized Gold is abandoned as money, whether through Economic Abandonment or Institutional Abandonment.

Let T_E denote the first realized state at which the joint configuration of Capitalism with Idealized Gold ceases to exist.

Then:

$$\boxed{T_E = \min\{T_A, T_V, T_R\},}$$

where the minimum is taken over termination events that are realized on the trajectory.

The expression excludes $T_{\min}$, which marks the exhaustion of Gold's internal ability to compensate further technical cheapening through lower physical productivity. Reaching $T_{\min}$ alone does not end the Idealized-Gold model.

If $T_A < \min\{T_V, T_R\}$, the model ends through abandonment of Idealized Gold.

Capitalism may continue with another monetary form.

If $T_V < \min\{T_A, T_R\}$, the model ends through Extinction of Valorization while Idealized Gold is still operative.

If $T_R < \min\{T_A, T_V\}$, the model ends through Reproductive Breakdown while Idealized Gold is still operative.

Simultaneous events are also possible.

Thus, either Idealized Gold is abandoned, or capitalism with Idealized Gold reaches terminal failure.

The first alternative may preserve capitalism by replacing the monetary form.

The second terminates the capitalist reproduction or valorization process represented by the model.

In either case, Idealized Gold does not survive as a stable terminal monetary form of the capitalist configuration analyzed here.

36. Refrigerator Theorem and Final Result

The preceding analysis distinguishes two levels of result.

The first is the internal **Refrigerator Dilemma** conditional on continued use of Idealized Gold.

The second is the termination of the complete **Capitalism-with-Idealized-Gold** configuration.

On the continued-Idealized-Gold branch, the labour mechanism yields:

$$N - \ell^*(t) \xrightarrow{\mathrm{E}} [0_E]_{\mathrm{T}}.$$

Repeated successful compensation therefore gives:

$$N - \ell(t) \xrightarrow{\mathrm{E}} [0_E]_{\mathrm{T}},$$

or crosses the positive-Surplus-Value boundary directly.

Consequently:

$$g(t), \bar{g}(t) \xrightarrow{\mathrm{E}} [0_E]_{\mathrm{T}}.$$

This is the Valorization Boundary T_V.

At the same time, the reproductive mechanism gives:

$$M_R(t) = x_R(t) - x(t),$$

with:

$$M_R(t) \downarrow_E .$$

The corresponding Transit Economic Boundary is:

$$M_R \xrightarrow{\mathrm{E}} [0]_{\mathrm{C}}.$$

Actual Reproductive Breakdown occurs only after crossing, $M_R(T_R) < 0$, equivalently $Y_A(T_R) < D$.

Thus, if Idealized Gold remains operative, the exhaustive orderings are $T_V < T_R$, $T_R < T_V$, or $T_V = T_R$.

There is no ordering on this continued-Gold branch in which the system preserves indefinitely both economically positive capitalist valorization and the necessary material reproduction of the economy.

From this follows the **Refrigerator Theorem**:

Theorem 1 (Refrigerator Theorem)**.** *The capitalist production of a freely reproducible money commodity has no stable positive trajectory on which both positive capital valorization and the necessary material reproduction are simultaneously preserved.*

The theorem concerns the internal trajectory on which the freely reproducible money commodity remains operative.

It does not claim that capitalism cannot escape the trajectory by abandoning that monetary form.

The broader result follows by combining the Refrigerator Theorem with the possibility of monetary abandonment.

Corollary 3 (Termination of the Idealized-Gold Model)**.** *Within the model, Capitalism with Idealized Gold cannot persist as a stable terminal configuration. If Idealized Gold remains operative as money, the system reaches either the Terminal Economic Boundary of positive capitalist valorization or Reproductive Breakdown, with simultaneous realization also possible. If Idealized Gold is abandoned economically or institutionally before such a terminal event, the Idealized-Gold model ends through the abandonment of its money commodity.*

The corollary distinguishes monetary exit from internal terminal failure. Economic or Institutional Abandonment ends the Capitalism-with-Idealized-Gold configuration but may leave capitalism operating under another monetary form; Extinction of Valorization and Reproductive Breakdown are terminal failures of the capitalist process represented within the model.

The internal Refrigerator mechanism rests on Monetary Character and Unlimited Reproducibility: Direct Monetary Realization permits expansion of Gold production without an independent natural scarcity boundary specific to the money commodity. Current Reproduction Value and Monetary Persistence also govern the technical path through Compensating Technical Deterioration, $T_{\min}$, and possible Technical Devaluation. If Technical Devaluation leads to abandonment, the monetary configuration ends; otherwise the internal Refrigerator Dilemma remains operative.

Part IX. Historical and Contemporary Cases

37. Gold Rushes

The distinction between Real Gold and Idealized Gold does not depend upon mathematical infinity. Idealized Gold need not exist in a literally infinite quantity. What matters is whether the natural limitation of its reproduction becomes economically binding on the scale of the society under consideration.

A finite deposit may therefore function as Idealized Gold if it is sufficiently large, accessible, and reproducible relative to the population, capital, and time horizon involved. A very large and relatively uniform gold deposit confronted by a sufficiently small society may remain effectively unrestricted for the entire economically relevant period.

Real Gold differs where the natural conditions of production begin to constrain further expansion. Richer deposits are exhausted, less productive deposits must be worked, extraction becomes more difficult, or further expansion under the previous conditions becomes impossible.

Historical gold rushes show how rapidly production can be reorganized when gold becomes exceptionally accessible.

Marx recorded an early example in *A Contribution to the Critique of Political Economy.* Citing an older historical account, he described an episode south of Prague in the year 760 in which unusually rich gold-bearing sands attracted large numbers of people away from agriculture. According to the account, the movement of labour was extensive enough that famine followed in the next year (Marx 1970).

The Australian gold rush of the 1850s provides a much better documented case. Following the discoveries of 1851, labour moved rapidly toward the goldfields, particularly in Victoria. In 1852, the male population of Tasmania fell by approximately 17 percent and that of South Australia by approximately 3 percent as people moved toward the gold-producing colonies. At the peak of the boom in 1852, mining accounted for approximately 35 percent of Australian GDP (Battellino 2010).

The effect was immediately visible in the labour market. Between 1850 and 1853, wages in Victoria rose by approximately 250 percent. Employers outside mining found it increasingly difficult to retain workers, and the wool industry was among the sectors directly affected (Battellino 2010).

Agriculture contracted especially sharply. Anderson estimates that agriculture's share of Australian GDP fell from approximately 34 percent to 16 percent within a single year after the discoveries in Victoria and New South Wales, as rural workers moved toward the goldfields (Anderson 2024).

The Australian case therefore shows a direct reallocation of labour toward Gold and away from ordinary production. Gold became sufficiently accessible and sufficiently attractive to reorganize a substantial part of the productive structure.

The process was later modified by conditions external to the goldfields themselves. Large-scale immigration expanded the labour force, the Australian population almost trebled over the following decade, and ordinary production expanded in response to the growing population and demand. At the same time, the Victorian gold rush itself declined substantially by the mid-1860s (Battellino 2010).

California provides another clear case.

Following the discoveries beginning in 1848, labour and capital moved into mining on such a scale that historical accounts describe crop cultivation during the early Gold Rush as having been brought to a near standstill. Agriculture subsequently faced persistent labour shortages through much of the 1850s and 1860s (Rawls and Orsi 1999).

California initially compensated for this shift through external trade. Food, clothing, construction materials, and other commodities were imported in large quantities. Even flour was initially imported before local agriculture expanded (Rawls and Orsi 1999).

The Gold sector therefore could absorb a very large proportion of local productive activity while part of the material requirements of the population were supplied by production performed elsewhere.

The natural conditions of Gold production later changed. The richest and most accessible surface deposits were progressively exhausted. Mining shifted toward more difficult and more capital-intensive forms, while some miners returned to agriculture or entered other activities. By 1853, supplying food to miners had already become a more stable livelihood for many former gold seekers (Rawls and Orsi 1999).

Australia and California show the same basic tendency. When Gold becomes unusually abundant relative to the productive scale of the society exploiting it, labour and capital can move toward Gold on a very large scale. Ordinary production then comes under pressure because the same productive resources cannot simultaneously remain fully employed elsewhere.

The movement does not continue indefinitely in the historical cases because the conditions of Real Gold eventually reassert themselves. Rich deposits are exhausted, extraction conditions worsen, immigration changes the scale of the labour force, and trade allows material requirements to be supplied from outside the local Gold-producing economy.

A sufficiently large discovery can therefore make Real Gold temporarily approximate Idealized Gold. The difference appears when the natural limitation of the deposit becomes economically effective.

38. Bitcoin as Digital Real Gold

Bitcoin provides a modern example of the distinction between Real Gold and Idealized Gold in a non-metallic form.

Bitcoin is produced through Proof of Work. The creation of new units is therefore connected to an actual expenditure of computational equipment, electricity, and labour. Nakamoto explicitly compared this process with gold mining: resources are expended in order to add new monetary units to circulation (Nakamoto 2008).

The decisive feature, however, is that greater expenditure on mining does not allow the Bitcoin sector to expand monetary production without limit.

Bitcoin regulates the difficulty of Proof of Work in response to changes in the computational power devoted to mining. If blocks are produced too rapidly, mining difficulty increases. Additional miners or more powerful equipment therefore increase competition for the existing flow of block rewards rather than allowing the aggregate rate of monetary production to rise proportionally (Nakamoto 2008).

The monetary supply is constrained independently of the amount of capital seeking to enter mining. New bitcoins are issued according to a predetermined schedule, the block subsidy is periodically reduced, and under the existing protocol the total supply approaches a limit of approximately 21 million bitcoins (Bitcoin.org 2026).

If the profitability of Bitcoin mining increases, additional capital can enter mining. More mining equipment can be purchased, more electricity can be consumed, and more labour can be employed. But this additional capital does not acquire the ability to reproduce the monetary object freely. The protocol responds by intensifying competition for a restricted monetary output.

In the terminology of the present work, Bitcoin therefore constitutes a form of **Digital Real Gold**. Its scarcity is imposed by the technical rules governing its reproduction.

Metallic Real Gold (that is gold itself) is limited by the natural conditions of deposits. Bitcoin is limited by protocol. In both cases, the reproduction of the monetary object encounters a constraint that cannot be removed merely because additional capital wishes to enter its production.

This feature is not incidental to Bitcoin's construction. Nakamoto combined costly production with controlled issuance from the beginning. The original Bitcoin paper specifies both Proof of Work and an adjustment of mining difficulty as computational power changes, and it anticipates a predetermined quantity of newly issued coins after which miners can be compensated through transaction fees rather than continued monetary expansion (Nakamoto 2008).

A digital monetary commodity could retain costly Proof of Work while removing the independent restriction on issuance. If twice as much capital and computational power entered mining and could consequently produce twice as many monetary units under unchanged conditions, the fact that mining remained costly would not solve the problem examined in this work. Costly production and restricted reproduction are separate properties.

Bitcoin combines both properties: Proof of Work gives production a real resource cost, while the issuance rules prevent additional capital from freely converting additional productive expenditure into an arbitrarily larger quantity of the monetary object.

39. Fan Tokens, Idealized Gold and Ideal Gold

The distinction between Real Gold and Idealized Gold can be clarified through the concept of a **Fan Token**: a monetary unit whose reproduction is not independently constrained. Game currencies provide a simple example: coins, credits, points, or similar units may perform monetary functions within a system while remaining reproducible whenever that system permits their creation.

For a money commodity to constitute **Real Gold** in the terminology of this work, two properties are essential. It must possess value as a produced commodity, and its reproduction must be independently constrained.

As shown in the preceding chapter, both **Material Real Gold** and **Digital Real Gold** possess these properties.

Commodity Value alone is therefore insufficient. A monetary object may require labour, machinery, electricity, or other real productive expenditure and therefore possess Commodity Value while remaining reproducible by additional capital without an independent reproductive constraint. Such an object is a **value-bearing Fan Token**, not Real Gold. Idealized Gold occupies this position: it retains Commodity Value and Monetary Character while the independent restriction on its reproduction has been removed. The Refrigerator Problem concerns this combination.

A further class of Fan Tokens does not require Commodity Value before entering circulation: **Fiat Money**. Modern fiat and credit money, together with the financial structures through which it is created, circulated, expanded, and extinguished, therefore raise a different problem from the one examined in this work. These monetary units are neither Real Gold nor Idealized Gold as defined here because they do not themselves carry Commodity Value.

This work has examined what happens when Gold retains Commodity Value but loses its independent reproductive constraint. *The Forms of Gold, Work III: Ideal Gold* examines monetary forms whose monetary units do not themselves carry Commodity Value.

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