

The Forms of Gold. Work I: Real Gold

Real Gold as Commodity Money and Capitalist Reproduction

Vilen Isteni
vilen@polarmarxism.com
polarmarxism.com

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Abstract

This paper develops the concept of Real Gold within Polar Marxism by examining whether commodity money can provide the monetary conditions required for the continuous expanded reproduction of aggregate social capital. Beginning from Capital as Self-Expanding Value, the analysis formulates Closure as the reproduction within each cycle of the conditions required for the next and adopts the existence of a Fully Consistent Closure as the strongest initial concession. A fixed-gold monetary benchmark is then progressively concretized by introducing finite circulation, hoarding, the value of the monetary commodity, and the production of additional monetary material. This yields a distinction between the required and actual quantity of Real Gold, its required and actual unit value, and the timing of its availability. Because the production and value of Real Gold are determined by their own material conditions, the monetary requirement of expanded reproduction does not itself generate the quantity, value, or timing needed for Closure. The paper distinguishes three possible regimes of adjustment: Real Gold, Accidental Perfect Adjustment, and Idealized Gold. Under Real Gold, Closure cannot be sustained because no mechanism ensures the required correspondence between the monetary needs of expanded reproduction and the actual movement of the monetary commodity. Under Accidental Perfect Adjustment, Closure could in principle occur if the required correspondence were reproduced by pure coincidence, but such a continuous coincidence is impossible. The remaining possibility is Idealized Gold, whose analysis is reserved for the subsequent work.

Keywords: Polar Marxism; Real Gold; commodity money; Capital; expanded reproduction; monetary realization; Closure; gold production; Gold Adjustment Problem; Idealized Gold

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Part I.

Capital and Monetary Realization

1. Capital as Self-Expanding Value

In this work, *Capital* is defined, following Marx, as *Self-Expanding Value*: value that reproduces itself through its own movement as an increased value Marx 1976.

The movement of Capital is continuous. Expanded value becomes the starting point of further movement, so each subsequent cycle continues an already existing process of self-expansion.

At the systemic level, Capital is characterized by *Sovereignty*. Sovereignty denotes a condition in which the self-expansion of value determines the movement of social reproduction as a whole. Production and the allocation of social resources take place within a system governed by the continued reproduction and expansion of Capital.

A second property is *Historical Continuity*. Capital in each subsequent cycle emerges from the result of its preceding movement:

$$K_t \longrightarrow K_{t+1}.$$

Here, K_t denotes Capital in the current cycle of reproduction, while K_{t+1} denotes Capital arising from its continued movement.

Within *Polar Marxism*, a system in which Self-Expanding Value possesses Sovereignty and Historical Continuity at the level of social reproduction is defined as *Center/Imperialism*. In the present work, Capital is examined at this systemic level.

2. The Law of Profit / The Law of Capital

As defined in Section 1, Capital exists as Self-Expanding Value. From this property follows the *Law of Profit / Law of Capital*:

Capital must valorize itself.

Capital must undergo valorization. Otherwise, value does not reproduce itself as Capital in the sense defined above.

Formally, this movement can be expressed as

$$V \longrightarrow V', \quad V' = V + \Delta V, \quad \Delta V > 0,$$

where V denotes the initial value, V' the value after valorization, and ΔV the increment of value.

3. The Monetary Form of Valorization

As established in Section 2, Capital must valorize itself. In capitalist movement, this process takes the *Monetary Form of Valorization*:

$$M - C \dots P \dots C' - M'$$

where M denotes the initial monetary value, C the commodities and conditions of production purchased by Capital, P the production process, C' the commodity capital containing increased value, and M' that same value after its monetary realization Marx 1976.

Valorization contains two successive moments. Increased value must first be produced:

$$C \dots P \dots C'.$$

Its existence in commodity form does not complete the movement of Capital. It must undergo *Realization*, that is, assume monetary form:

$$C' \longrightarrow M'.$$

Accordingly, C' and M' represent distinct stages of the same movement:

$$C' \neq M'.$$

Capital must first produce increased value and then realize it in money. Only with the emergence of M' is the cycle of Valorization completed, and M' becomes the initial monetary form of the next cycle of Capital.

4. Individual Capital and Aggregate Social Capital

As established in Section 3, the movement of Capital is completed only through the transformation of C' into M' . At the level of *Individual Capital*, this condition applies to an individual circuit:

$$C'_i \longrightarrow M'_i.$$

Here, the index i denotes an individual capital.

Aggregate Social Capital is the totality of individual capitals whose movements form a unified process of social reproduction Marx 1978.

The realization of an individual capital presupposes a buyer, since the receipt of M'_i by one capital corresponds to monetary expenditure by another party. The successful realization of an individual C'_i therefore does not by itself establish the possibility of realizing aggregate commodity capital.

At the systemic level, the *Realization Problem / Buyer Problem* arises from the need to establish the source of purchasing power through which the expanded aggregate commodity capital is monetarily realized in each cycle of expansion.

$$\sum_i C'_i \longrightarrow \sum_i M'_i.$$

Whenever Capital is considered at the systemic level in what follows, it refers specifically to *Aggregate Social Capital*. This corresponds to the level of analysis established in Section 1.

5. Expanded Reproduction and Closure

As follows from Section 2, the continued movement of Capital presupposes its further self-expansion. Its reproduction therefore takes the form of *Expanded Reproduction*, in which the result of the current cycle becomes the basis of the next cycle on an enlarged scale Marx 1978:

$$K_{t+1} = K_t + \Delta K_t, \quad \Delta K_t > 0.$$

Here, K_t denotes Capital in the current cycle, while ΔK_t denotes its increment carried into the next cycle.

Closure denotes a reproduction of Aggregate Social Capital in which the current cycle generates, within the system itself, the conditions required for the beginning of the next expanded cycle.

Closure requires the current cycle to reproduce the material conditions of the next one, including labor power, the replacement of worn-out means of production, the means required for the expansion of production, and the monetary realization of the commodity capital produced. The structure of the means of production actually produced must also correspond to the structure required for the expansion of productive capacity.

This material correspondence can be expressed as

$$\mathbf{I}_t^{\text{produced}} = \mathbf{I}_t^{\text{required}},$$

where $\mathbf{I}_t^{\text{produced}}$ denotes the vector of investment goods actually produced, while $\mathbf{I}_t^{\text{required}}$ denotes the vector of means of production required for the expansion of productive capacity in the next cycle.

Equality between aggregate monetary magnitudes does not by itself establish Closure. For example,

$$\sum_i \Pi_i = \sum_i I_i$$

expresses equality between aggregate profit and aggregate investment, but does not determine whether particular capitals obtain the specific means of production and monetary funds required for the continuation of Expanded Reproduction.

Part II.

The Closure Problem and the Fixed-Gold Benchmark

6. The Monetary Benchmark

For the analysis of Closure, we introduce a *Monetary Benchmark*: a closed capitalist system in which gold serves as the monetary form.

The total physical stock of gold in the system is fixed:

$$G_t = \bar{G},$$

where G_t denotes the physical quantity of gold in period t , while $\bar{G}$ denotes the constant total stock of gold.

Gold functions as *Commodity Money* and retains its own value as a commodity. The condition $G_t = \bar{G}$ therefore refers to the physical mass of gold rather than to an abstract quantity of monetary units.

The model contains neither *Credit* nor *Fiat Money*. Monetary realization must occur through the existing stock of gold. Barter and other forms that bypass monetary realization are likewise excluded.

The same unit of gold may participate successively in multiple transactions. A fixed stock of gold therefore does not imply a fixed volume of monetary circulation. Let V_t denote the *Velocity of Circulation*.

At this level of abstraction, no finite physical upper bound is imposed on V_t . If Closure requires a higher velocity of circulation, the model permits V_t to increase to whatever magnitude is formally required.

The Monetary Benchmark therefore combines a fixed physical stock of gold with the strongest possible capacity of that stock to sustain monetary circulation. The possibility of Closure will be examined within this benchmark.

7. The Closure Fork: S or Not-S

Within the Monetary Benchmark, the question of Closure takes the form of a logical fork:

$$\boxed{S \vee \neg S}.$$

S denotes the existence of a *Fully Consistent Closure* in which Aggregate Social Capital is capable of continuously carrying out Expanded Reproduction under all the conditions established in Part I and Section 6.

In other words, S asserts the existence of a movement of the system in which each cycle is simultaneously completed through monetary realization and reproduces the conditions required for the next expanded cycle.

$\neg S$ denotes the absence of such a fully consistent closure under the same initial conditions.

At this stage, neither S nor $\neg S$ is treated as proven. Different models may demonstrate particular forms of coordination within reproduction, but the existence of a coordinated result still requires a mechanism through which that result is reproduced from cycle to cycle.

8. The Debate over Closure

The question of whether Closure is possible has a long history in theories of capitalist reproduction. Tugan-Baranovsky regarded *Expanded Reproduction* as possible within the capitalist system itself, provided that the necessary proportions between sectors were maintained Tugan-Baranovsky 2002.

Luxemburg took the opposite position. In *The Accumulation of Capital*, she argued that expanded accumulation could not be fully realized within the capitalist system alone and therefore required a non-capitalist environment Luxemburg 2003.

Bauer and Bukharin, by contrast, defended the possibility of internal Expanded Reproduction within a closed capitalist system Bauer 1986; Bukharin 1972.

Grossmann continued the analysis of reproduction schemes, treating them as an abstract level of analysis that required further concretization Grossmann 1992.

Later, Morishima and Roemer examined reproduction, growth, and equilibrium through formal economic models Morishima 1973; Roemer 1978, 1981. Nikaido separately analyzed the mechanism of capital movement between sectors and its relation to equilibrium Nikaido 1983, 1985. Satoh continues the formal analysis of reproduction by connecting the movement of capital with a multi-sector model Satoh 2026.

Historically, the debate has therefore included positions compatible with S as well as positions compatible with $\neg S$. At this stage, the present work adopts neither position and does not enter into a polemic with these authors.

9. Equilibrium and Real Reproduction

Although Section 8 stated that the present work does not enter into a polemic with the participants in the historical debate, the requirement implied by S itself must be specified. A position asserting the existence of S must establish not only the possibility of a coordinated state of the system, but also the mechanism through which such a state arises and is reproduced in its actual movement.

Existence of Equilibrium and *Competitive Convergence* refer to different questions. The formal existence of a state satisfying the conditions of Expanded Reproduction does not establish that the movement of Capital can generate and reproduce that state.

Actual reproduction proceeds through decisions of capitals responding to differences in profitability, altering investment, and reallocating productive capacity. A time lag therefore exists between the relevant signal and the corresponding change in production:

$$\pi_{i,t} \longrightarrow I_{i,t} \longrightarrow K_{i,t+\tau} \longrightarrow Q_{i,t+\tau}, \quad \tau > 0.$$

Here, $\pi_{i,t}$ denotes profitability in sector i , $I_{i,t}$ the investment induced by it, $K_{i,t+\tau}$ productive capital after the adjustment period, and $Q_{i,t+\tau}$ the corresponding level of production.

The existence of such a lag does not by itself establish the impossibility of Closure. Depending on the structure of capital's response, emerging deviations may contract or expand. S therefore requires a

mechanism capable of reproducing the necessary proportions through the actual movement of the system.

The dynamics are also subject to the *No-Restart Condition*. Each new cycle begins from the state produced by the preceding one:

$$X_{t+1} = F(X_t, a_t, \varepsilon_t),$$

where X_t denotes the actual state of the system, a_t the actions of capitals, and ε_t the deviations and disturbances that arise. Existing productive capacity, infrastructure, inventories, and the results of past investment are carried into the next cycle and constrain subsequent movement.

Closure requires *Endogenous Recovery* from emerging deviations. Even a high probability of successfully completing an individual cycle does not guarantee sustained reproduction across a long sequence of cycles. If the probability of successfully completing one cycle is independently given by $p < 1$,

$$P_n = p^n,$$

where P_n denotes the probability of completing n consecutive cycles without violating the specified condition.

10. Assume S

For the purposes of the further analysis, we assume S , which, under the conditions established in the preceding sections, means that *Fully Consistent Closure* exists and that Aggregate Social Capital is capable of continuously carrying out Expanded Reproduction.

If $\neg S$ were assumed instead, the analysis of Closure at this level would end, since closed Expanded Reproduction under the established conditions would be impossible.

S is therefore adopted as the *Strongest Concession* to the system under consideration.

The productive side of Closure is thereby fixed. The analysis now turns to the monetary side and to the properties of gold as a monetary commodity.

Part III.

From Fixed Gold to Real Gold

11. Commodity Money and Material Unity

After assuming S , the analysis returns to the monetary side of the movement of Capital. As established earlier, produced value must undergo monetary realization for the movement of Capital to continue. Money therefore becomes the point through which the further movement of value must pass within capitalist reproduction.

The monetary object examined in the present work will be referred to as *Real Gold*. Real Gold does not mean literal physical gold. It is a broader category of monetary commodity that simultaneously functions as money and possesses its own commodity value. For simplicity, Real Gold may be understood as Marxian *Commodity Money* extended beyond the specific case of material gold analyzed by Marx. Within Polar Marxism, this extended category of commodity money is termed Real Gold.

The contribution of Marx is particularly important here. His analysis of material gold as commodity money clearly established the unity of these two sides within a single monetary commodity Marx 1976.

In addition to G_t , introduced in Part II, we introduce

$$v_{G,t}$$

where $v_{G,t}$ denotes the value of one unit of Real Gold in period t .

12. Finite Circulation

In Section 10 of Part II, we assumed S . However, S was assumed within the Monetary Benchmark, where the *Velocity of Circulation* could take whatever magnitude was required for Closure. The transition to *Real Gold* requires this assumption to be removed and the actual properties of the monetary commodity to be taken into account.

Every form of Real Gold possesses a finite Velocity of Circulation. For any given monetary commodity, there exists some finite $V_{\max}$:

$$V_t \leq V_{\max} < \infty.$$

The specific magnitude of $V_{\max}$ depends on the form of Real Gold. In the case of physical gold, circulation requires the transfer of a material object and therefore proceeds at a particularly low velocity. Historical estimates of monetary circulation during the gold-standard era are of the order of only several turnovers per year National Bureau of Economic Research 2012. For the purposes of the present work, physical gold is therefore taken to satisfy

$$V_t < 10.$$

Other forms of Real Gold may possess substantially higher velocities of circulation. A digital monetary commodity, or *Digital Real Gold* (to be discussed in subsequent works), may circulate faster

than physical gold. Its $V_{\max}$ may lie at a higher finite order 10^k , where k is determined by the technical and institutional conditions of the corresponding form of Real Gold.

For the subsequent analysis, let T_t denote the monetary volume of turnover that must be serviced in period t . Since G_t denotes the physical quantity of Real Gold and $v_{G,t}$ the value of one unit of Real Gold, the required velocity of circulation is

$$V_t^{\text{required}} = \frac{T_t}{v_{G,t}G_t}.$$

Within the Monetary Benchmark, the physical stock of gold is fixed:

$$G_t = \bar{G}.$$

If monetary turnover grows together with Expanded Reproduction at a constant positive rate g ,

$$T_t = T_0(1 + g)^t, \quad g > 0,$$

then, holding the value of one unit of Real Gold constant,

$$v_{G,t} = \bar{v}_G,$$

the required velocity of circulation becomes

$$V_t^{\text{required}} = \frac{T_0}{\bar{v}_G \bar{G}}(1 + g)^t$$

and therefore

$$V_t^{\text{required}} \longrightarrow \infty \quad \text{as} \quad t \longrightarrow \infty.$$

Finite Circulation therefore excludes an arbitrarily increasing V_t as a permanent mechanism for sustaining Closure under a fixed monetary stock. Once this constraint is introduced, further adjustment must occur through $v_{G,t}$ and G_t .

13. Hoarding and Monetary Reserves

Not all existing Real Gold participates in circulation at the same time. Part of the monetary commodity is held in the form of *Hoards and Monetary Reserves* Marx 1978.

The total stock of Real Gold is therefore divided into circulating and reserve components:

$$G_t = G_t^{\text{circ}} + H_t,$$

where G_t^{circ} denotes the quantity of Real Gold directly participating in circulation, while H_t denotes the quantity held outside current circulation.

Since H_t constitutes a part of the total existing stock of Real Gold, it is bounded by G_t :

$$0 \leq H_t \leq G_t.$$

As established in Section 12, the Velocity of Circulation lies within a finite range:

$$0 < V_t < 10^k.$$

For the present analysis, let some given value $\bar{V}$ within this range be fixed:

$$V_t = \bar{V}.$$

At a given $\bar{V}$, monetary turnover T_t can be serviced only by the circulating part of the monetary stock:

$$T_t \leq \bar{V} v_{G,t} G_t^{\text{circ}}.$$

Since

$$G_t^{\text{circ}} = G_t - H_t,$$

it follows that

$$T_t \leq \bar{V} v_{G,t} (G_t - H_t).$$

Hoarding reduces the quantity of Real Gold directly available for current monetary realization. At the same time, H_t constitutes a reserve that may return to circulation in subsequent periods.

14. The Required Monetary Stock

From the condition of monetary realization,

$$T_t \leq \bar{V} v_{G,t} (G_t - H_t),$$

follows the minimum *Required Monetary Stock*:

$$G_t^{\text{required}} \geq \frac{T_t}{\bar{V} v_{G,t}} + H_t.$$

Here, G_t^{required} denotes the minimum quantity of Real Gold required to service monetary turnover T_t at given $\bar{V}$, $v_{G,t}$, and H_t .

Under Expanded Reproduction, growth in T_t increases G_t^{required} if the remaining parameters are held constant. Adjustment may occur through changes in $v_{G,t}$, H_t , or G_t itself.

Since $\bar{V}$ is already bounded and H_t cannot continuously decline below zero, continued expansion eventually raises the question of increasing the stock of Real Gold itself. At this point, the fixed condition $G_t = \bar{G}$ established in the Monetary Benchmark is no longer sufficient for the subsequent analysis.

15. The Remaining Adjustment Variables

As established in Section 13, Hoards and Monetary Reserves are bounded by the existing stock of Real Gold:

$$0 \leq H_t \leq G_t.$$

Thus, H_t may temporarily alter the quantity of Real Gold participating in circulation, but it cannot serve as an unlimited source of monetary adjustment. In the long-run analysis, changes in reserves represent fluctuations within the already existing G_t .

The Velocity of Circulation V_t also remains a variable magnitude. It may increase or decrease within the finite range established earlier,

$$0 < V_t < 10^k,$$

but its fluctuations cannot provide unlimited adjustment to continuously growing monetary turnover.

The principal remaining variables of monetary adjustment are therefore the physical quantity of Real Gold G_t and the value of one unit of Real Gold $v_{G,t}$.

Consider first what would be required of $v_{G,t}$ if G_t did not increase. At given G_t , H_t , and V_t , the condition of monetary realization requires

$$v_{G,t}^{\text{required}} \geq \frac{T_t}{V_t(G_t - H_t)}.$$

If Expanded Reproduction is accompanied by a constant positive growth of monetary turnover,

$$T_t = T_0(1 + g)^t, \quad g > 0,$$

then, with the remaining magnitudes unchanged, the required $v_{G,t}$ must increase together with T_t . An increase in the value of one unit of Real Gold implies a corresponding decline in commodity prices expressed in Real Gold:

$$v_{G,t} \uparrow \implies P_t \downarrow.$$

With a fixed G_t , the system would therefore require a definite deflationary trajectory. Under exponential growth of T_t , the required $v_{G,t}$ would also have to increase systematically, while the price level would have to decline continuously.

It is therefore necessary to distinguish between

$$v_{G,t}^{\text{required}} \quad \text{and} \quad v_{G,t}^{\text{actual}}.$$

The former is determined by the requirements of monetary realization. The latter is the actual value of Real Gold and changes under the real conditions governing the production and circulation of the monetary commodity. No identity between these two trajectories has yet been established.

In long-run economic practice, the general price level is normally associated with relative stability or low positive inflation. The Federal Reserve and the European Central Bank, for example, maintain

inflation objectives of approximately 2% over the longer or medium term, with the ECB explicitly defining this objective as consistent with price stability Board of Governors of the Federal Reserve System 2025; European Central Bank 2025. This differs fundamentally from the trajectory required under a fixed G_t , in which prices would have to decline continuously from cycle to cycle as T_t expands.

Such a deflationary trajectory cannot be produced by the requirement of Closure itself. For $v_{G,t}^{\text{actual}}$ to increase continuously, real processes must continuously increase the value of one unit of Real Gold. In the case of material gold, within the Marxian theory of value, this would require a systematic increase in the socially necessary labor required to produce a unit of gold relative to the production of other commodities Marx 1976. The conditions of gold production would therefore have to deteriorate continuously at the rate required to sustain the necessary deflation.

Such a movement does not follow from Expanded Reproduction itself. The Monetary Requirement determines which $v_{G,t}$ would be necessary for Closure, but it does not create a productive mechanism that forces $v_{G,t}^{\text{actual}}$ to assume that value. Deflationary adjustment through the value of Real Gold therefore itself requires a separate mechanism.

Thus, $v_{G,t}$ remains one of the two principal objects of the subsequent analysis, but it cannot simply be assumed to take the required value automatically. The second object remains

$$G_t.$$

Part IV.

The Production of Real Gold

16. The Reproduction of Money Material

Once the condition of a fixed monetary stock $G_t = \bar{G}$ is removed, the quantity of Real Gold becomes a variable magnitude:

$$G_{t+1} = G_t + \Delta G_t,$$

where ΔG_t denotes the net change in the quantity of Real Gold between periods t and $t + 1$.

Reproduction of Money Material denotes the process through which an additional quantity of the monetary commodity is produced, enters circulation, and increases the existing G_t Friedenthal 2021; Grossmann 1932; Marx 1978.

If the continuation of Expanded Reproduction requires

$$G_{t+1}^{\text{required}} > G_t,$$

then the monetary side of Closure requires a positive increment of monetary material:

$$\Delta G_t > 0.$$

The production of Real Gold thereby becomes an independent condition of the continued reproduction of Aggregate Social Capital. The monetary material must now not only exist and circulate, but also be produced.

17. Gold Production as a Capitalist Sector

A positive increment of Real Gold,

$$\Delta G_t > 0,$$

must be produced. In the case of material gold, this implies the existence of a *Gold-Producing Sector* functioning as part of capitalist production.

Let $K_{G,t}$ denote the capital advanced to gold production. This capital is spent on labor power, means of production, energy, and the other elements required by the production process.

However, an equal magnitude of $K_{G,t}$ does not necessarily produce an equal quantity of gold. The result also depends on the *Material Conditions of Gold Production*: the accessibility of deposits, the gold content of ore, geological conditions, the attained level of productivity, and the time required to expand extraction.

The production of new Real Gold can therefore be represented in general form as

$$\Delta G_t = \Phi(K_{G,t}; \Gamma_{G,t}, \tau_G),$$

where $K_{G,t}$ denotes the capital directed into gold production, $\Gamma_{G,t}$ denotes the totality of the material conditions of gold production, and τ_G denotes the production lag.

An increase in G_t therefore requires the allocation of capital to a distinct productive sector and takes place within the material constraints of that sector. ΔG_t is consequently the result of real production rather than a freely adjustable monetary magnitude.

18. Gold and Aggregate Reproduction

The Gold-Producing Sector does not exist outside Aggregate Social Capital. The capital directed into the production of Real Gold constitutes a part of total social capital:

$$K_t = K_{G,t} + K_{O,t},$$

where $K_{G,t}$ denotes the capital of the gold-producing sector, while $K_{O,t}$ denotes the capital of the remaining productive sectors.

An increase in $K_{G,t}$ therefore implies a change in the distribution of social capital.

If further monetary realization requires an increase in the stock of Real Gold,

$$G_{t+1}^{\text{required}} > G_t,$$

then a part of Aggregate Social Capital must be directed toward the expansion of the Gold-Producing Sector. The system's need for an additional monetary commodity therefore becomes a requirement to reallocate part of capital, means of production, and labor power from the production of other commodities into the production of Real Gold.

This reallocation arises because the continuation of Expanded Reproduction requires the monetary realization of produced value. A part of social production must therefore be directed toward producing the very means through which the remaining produced value is realized in monetary form.

However, an increase in $K_{G,t}$ does not imply that the required quantity of Real Gold will actually be produced. As established in the preceding section, the result of gold production depends not only on the magnitude of advanced capital but also on the material conditions of production. It is therefore necessary to distinguish between

$$\Delta G_t^{\text{required}} \quad \text{and} \quad \Delta G_t^{\text{actual}}.$$

The reallocation of additional capital into the Gold-Producing Sector may fail to produce

$$\Delta G_t^{\text{actual}} = \Delta G_t^{\text{required}}.$$

At the same time, $v_{G,t}^{\text{actual}}$ continues to change in accordance with the actual conditions of production of Real Gold. Even after the reallocation of social capital toward gold production, the system therefore acquires no automatic correspondence between the required quantity of the monetary commodity and its actual quantity or value.

19. Several Lines of Material Reproduction

After incorporating Real Gold into the analysis of Closure, several distinct lines of reproduction can be identified which must be coordinated simultaneously.

The first concerns the ordinary material structure of Expanded Reproduction. As established in Part I, the means of production actually produced must correspond to those required for the next cycle:

$$I_t^{\text{produced}} = I_t^{\text{required}}.$$

Equality in the aggregate magnitude of investment alone is insufficient. The corresponding means of production must exist in the corresponding sectors and at the corresponding time.

The second line concerns the quantity of Real Gold. The actually existing and produced monetary commodity must correspond to the quantity required by monetary turnover:

$$G_t^{\text{actual}} = G_t^{\text{required}}.$$

The third concerns the value of one unit of Real Gold:

$$v_{G,t}^{\text{actual}} = v_{G,t}^{\text{required}}.$$

Correspondence in the quantity of gold alone is insufficient, since the same physical mass of Real Gold represents a different monetary value at different $v_{G,t}$.

The aggregate value of the available monetary commodity therefore also matters:

$$M_{G,t}^{\text{actual}} = v_{G,t}^{\text{actual}} G_t^{\text{actual}}.$$

At a given Velocity of Circulation, the actual monetary stock must be capable of servicing the required monetary turnover. If Hoards and Monetary Reserves are taken into account, the monetary value directly available for circulation is

$$M_{G,t}^{\text{circ}} = v_{G,t}^{\text{actual}} (G_t^{\text{actual}} - H_t).$$

However, correspondence between these magnitudes over an entire period is still insufficient. The monetary commodity must be available precisely when the corresponding monetary realization has to occur. Closure therefore also requires temporal correspondence between the emerging need for money and the actual availability of Real Gold.

Aggregate Reproduction thus requires the simultaneous coordination of at least four dimensions: the structure of ordinary production, the quantity of Real Gold, the value of one unit of Real Gold, and the timing of its availability. A mismatch in any one of these lines is not eliminated by correspondence in the others.

Within S , we have already assumed that the ordinary material side of Expanded Reproduction is capable of achieving the necessary correspondence, including the required allocation of investment and means of production. No equivalent assumption has been made for the monetary side. No mechanism has yet been established that would systematically coordinate the continuously changing need for Real Gold with its actual quantity, value, and timing of appearance. Under Expanded Reproduction accompanied by a growing Monetary Requirement, Closure therefore requires not

merely the production of new Real Gold, but its production in the required quantity, with the required value, and at precisely the moment when the corresponding monetary realization must occur.

20. Monetary Waste

The production of Real Gold requires directing a part of Aggregate Social Capital into the Gold-Producing Sector. This capital employs labor power, means of production, and other material conditions that could otherwise participate in the production of other commodities.

Let the share of social capital directed into the production of Real Gold be denoted by

$$\omega_{G,t} = \frac{K_{G,t}}{K_t}.$$

As long as monetary realization requires the production of additional Real Gold,

$$\omega_{G,t} > 0.$$

This part of social production is necessary not because Real Gold directly expands the productive capacity of the remaining sectors or enters individual consumption, but because the monetary form of realization itself requires the reproduction of a distinct monetary commodity.

In this sense, the production of Real Gold constitutes *Monetary Waste*: a part of social labor and means of production must be continuously directed toward the production of the material bearer of monetary realization.

21. The Gold Adjustment Problem

The preceding analysis established the conditions that the monetary side of Closure must satisfy. At the required moment, the already familiar *Triad* must hold:

$$G_t^{\text{actual}} = G_t^{\text{required}},$$

$$v_{G,t}^{\text{actual}} = v_{G,t}^{\text{required}},$$

$$\tau_t^{\text{actual}} = \tau_t^{\text{required}}.$$

Until this point, Real Gold has been considered together with all of its real productive constraints. Its actual quantity is determined by the production of the monetary commodity, its actual value by the conditions of that production, and the moment at which additional Real Gold appears by the duration of the corresponding production process. The Monetary Requirement itself determines none of these magnitudes.

The direction of additional capital into the Gold-Producing Sector does not eliminate the problem. It does not guarantee

$$\Delta G_t^{\text{actual}} = \Delta G_t^{\text{required}},$$

does not establish the required $v_{G,t}^{\text{actual}}$, and does not ensure that the result appears at the required moment.

Consequently, under the regime considered so far, correspondence between actual and required magnitudes may occur, but no regularity has been established that would ensure such correspondence. Under Expanded Reproduction, the Monetary Requirement repeatedly arises and grows together with the expansion of reproduction, while the quantity, value, and timing of Real Gold continue to be determined by their own real conditions.

This allows us to identify the first of three possible regimes: *Real Gold*. Under this regime, the actual movement of the monetary commodity is not directly subordinated to the Monetary Requirement. Real Gold therefore does not provide a stable monetary Closure of Expanded Reproduction. As expanded reproduction continues, the monetary requirement continues to change, while no mechanism has been established that would permanently bring Real Gold into correspondence with it.

The second possible regime is *Accidental Perfect Adjustment*.

It may be assumed that no mechanism of adjustment exists, but that the actual movement of Real Gold nevertheless, by pure accident, repeatedly takes exactly the form required by the system; that is, the Triad is satisfied.

Real Gold remains Real Gold under this assumption. Its production continues to depend on material conditions, its value continues to change according to the real conditions of production, and its appearance continues to require time. The only additional assumption is that the outcomes of all these independent processes happen to coincide with the Monetary Requirement.

Such a coincidence is logically possible. In an individual cycle, the actual quantity of Real Gold produced may accidentally be exactly the quantity required, its value may accidentally assume the required magnitude, and the result may accidentally appear at precisely the required moment.

Expanded Reproduction, however, requires more than a single coincidence. The same correspondence must be reproduced in the next cycle, then again in the following cycle, and so on throughout the entire sequence of expanded reproduction, despite the continuously changing Monetary Requirement.

Accidental Perfect Adjustment is therefore not a mechanism of Closure. It represents a limiting case of Real Gold in which the independent actual movement of the monetary commodity accidentally coincides with the required trajectory. The longer the sequence of reproduction, the greater the number of consecutive coincidences that must be assumed. We therefore conclude that this is impossible, and that its probability tends toward zero.

The third possible regime is *Idealized Gold*.

The initial properties of the monetary object can now be altered. In the case of Real Gold, the direction of capital into the Gold-Producing Sector did not determine an exact result because production remained constrained by material conditions:

$$\Delta G_t = \Phi(K_{G,t}; \Gamma_{G,t}, \tau_G).$$

Suppose now that these constraints are removed to the extent that they prevent correspondence with the Monetary Requirement. If the system requires a particular additional quantity of monetary material, capital directed into its production can produce exactly that quantity. If a particular value of one unit is required, that value can likewise correspond to the requirement. If the monetary material is required at a particular moment, it can be available at that moment.

In other words, the systematic fulfillment of the Triad becomes possible.

Such a monetary object will be called *Idealized Gold*.

Idealized Gold differs from Accidental Perfect Adjustment because the correspondence between actual and required magnitudes is no longer assumed to be an accidental result of the independent movement of Real Gold. It becomes possible because the material constraints that, under Real Gold, prevented the movement of capital from directly determining the required result are removed. Capital can now be directed into the production of monetary material without the previous gap between the magnitude of advanced capital and the quantity, value, and timing of the monetary object actually obtained.

The introduction of Idealized Gold does not yet solve the problem of Closure. It creates a new theoretical object and new conditions under which the systematic fulfillment of the Triad becomes possible. The next question is how Capital will move after the removal of the constraints $\Gamma_{G,t}$, how this movement will affect Idealized Gold, and what all of this will ultimately lead to.

The analysis of the movement of Capital, its effect on Idealized Gold, and what this process ultimately leads to constitutes the subject of the next work, *Idealized Gold*. The analysis of Real Gold ends here.

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