The Joint Reproduction of the Nomenklatura and Its Party-State

A Formal Model of Systemic Growth and Discipline

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Abstract

This paper develops a formal model of the joint reproduction of the nomenklatura and the party-state. The nomenklatura is modeled as the command class-representative of an already constituted party-state, with its systemic position tied to developedness and institutional reproduction. Under positive-development conditions, this structure generates developmental alignment toward maximum accessible role-bound effort, while adaptive voice coordinates discipline around a finite system-optimal level. Relative developedness evolves against an advancing world frontier, linking catching-up development to the accumulation of material and organizational command capacity. Collapse-contingent market conversion is non-generalizable at the class-wide level, reinforcing reproduction-preserving systemic interest. When attainable calibrated development becomes non-positive, persistent relative erosion can deplete inherited command-capacity reserves, constrain disciplinary calibration, weaken strategic blocking, and raise the effort required for reproduction. The system then bifurcates between crisis containment and collapse according to the reproduction threshold. The central result separates systemic interest from historical capacity: the nomenklatura may continue to prefer and support party-state reproduction after the developmental and command conditions required to realize that interest have become unattainable.

Keywords: nomenklatura; party-state; systemic reproduction; developmental alignment; catching-up development; command capacity; socialist political economy; polar Marxism; anticapitalism; RDP
JEL Codes: P27; P30; D73; O11; H11

1 Introduction

This article provides a formal map of my theory of the joint reproduction of the nomenklatura and the party-state. It translates into an explicit model the set of assumptions, causal relations, and theoretical intuitions that underlies my broader account of the nomenklatura, systemic development, discipline, catching-up dynamics, and systemic crisis. Ideas that can be expressed substantively in a small number of theoretical propositions are unfolded here into a system of definitions, functions, constraints, dynamic transitions, assumptions, and formal results. The resulting scale of the model reflects the number of relations contained within a comparatively simple underlying logic.

Scope of the model. This article is intended as a working formalization of the theoretical logic underlying my account of the nomenklatura and the party-state. Its purpose is to make explicit the assumptions, causal relations, mechanisms, and regularities that are otherwise carried in substantive theoretical reasoning, and to place them in a form that can be inspected, referenced, revised, and extended. The model should therefore be read as a formal map of the theory in its present state rather than as a closed and fully specified empirical or structural model. Its role is to provide an explicit representation of the logic being advanced and a stable framework to which later refinements, extensions, and revisions can be attached.

The central purpose of the article is to make that logic explicit and internally traceable. The model collects in one place the mechanisms that stand behind particular propositions of the broader theory and thereby provides a reference structure to which those propositions can be referred back. The claim that the nomenklatura is structurally interested in development is represented through developmental alignment; the logic of catching-up development is represented through the law of motion of relative developedness; the systemic function of discipline is represented through the problem of disciplinary calibration; the relation between individual and class interest is represented through the distinction among individual conduct, hierarchically weighted class outcomes, and decisively weighted bodies; and the logic of systemic crisis is represented through developmental exhaustion, erosion of command capacity, and the reproduction threshold. The model therefore serves as a formal specification of the causal content carried by these theoretical claims.

The starting object is an already constituted party-state under second-type sovereignty and a nomenklatura that already exists within it as the system's command class-representative. The party-state constitutes the institutional form through which systemic development, administration, discipline, and reproduction are organized, while the nomenklatura performs the corresponding command function. Within my broader theory, this institutional configuration expresses the organization of absolute proletarian sovereign power. The reproduction of the nomenklatura is therefore linked directly to the reproduction of the party-state and to the system's capacity to realize development through that institutional form.

This relation generates the central developmental mechanism of the model. The systemic value of a nomenklatura position rises with the developedness of the party-state, while the developmental effort of individual agents enters aggregate effort and thereby contributes to the developedness and stability realized in the subsequent systemic state. Under positive-development conditions, this structure aligns the system-oriented nomenklatura with maximum accessible role-bounded developmental effort and, at the level of the reproduction-preserving body as a whole, with maximum accessible aggregate developmental effort. The model thereby formalizes the intuition that the nomenklatura's own systemic interest links it to the expansion of the productive, technological, organizational, and administrative capacities of the party-state.

Development has both an absolute and a relative dimension. The party-state develops in relation to an advancing world developedness frontier, and its relative position depends on the relationship between its own rate of development and the movement of that frontier. Growth above the frontier growth rate produces catching-up development and raises relative developedness. Equal growth preserves the proportional developmental position. Growth below the frontier rate produces relative developmental erosion. This structure formalizes catching-up development as a historical movement relative to a continuously advancing external frontier. Sustained catching-up periods also accumulate the material, organizational, and administrative foundations of future command capacity.

Discipline forms the second side of the same reproduction mechanism. It constrains deviation, supports governability, and carries its own systemic cost. The party-state therefore possesses a finite system-optimal disciplinary level that maximizes calibrated development. Nomenklatura agents retain heterogeneous private interests and disciplinary preferences, while their adaptive voice contains a common developmental component generated by their shared exposure to systemic development. At the aggregate level, this structure produces a restoring tendency around a potentially moving system-optimal disciplinary level and allows individual heterogeneity to coexist with a coherent developmental trajectory. Under positive-development conditions, maximum accessible developmental effort, calibrated discipline, sufficient disciplinary capacity, strategic blocking, and systemic reproduction form an integrated reproductive-growth regime.

The model also formalizes the class structure of the choice between party-state reproduction and collapse. Within a reproduced party-state, serious all-cause disciplinary displacement affects a structurally small share of the nomenklatura body. Collapse has a different institutional incidence: the inherited nomenklatura position ceases to be reproduced for the entire command body. Post-collapse market conversion begins only after that institutional loss and determines the subsequent individual positions of former nomenklatura agents. Successful conversion becomes progressively non-generalizable for the nomenklatura as a hierarchically weighted class. This structure supplies a material basis for the nomenklatura's systemic interest in party-state reproduction while preserving individual heterogeneity, private deviation, localized free riding, and exceptional post-collapse outliers.

The crisis mechanism develops from the same set of relations. Accumulated developedness and an inherited command-capacity reserve allow the party-state to continue reproducing through an initial period of relative developmental erosion. Exhaustion of attainable positive calibrated development moves the system into the exhausted-development regime. Continued relative erosion then places pressure on disciplinary capacity, consumes inherited command-capacity reserves, can separate realized discipline from its unconstrained systemic optimum, can open strategically relevant breach channels, and can raise the developmental effort required to preserve reproduction. The realized relation to the reproduction threshold determines the resulting branch: configurations that remain above the threshold produce crisis containment, while configurations that fall below it produce collapse.

A central result of the model is the separation between the nomenklatura's systemic interest and the historical capacity of the party-state to realize that interest. The nomenklatura may continue to prefer party-state reproduction, continue to supply maximum accessible developmental effort, and continue to generate disciplinary pressure toward the system-optimal level while the material and organizational conditions required for reproduction progressively become unattainable. Collapse can therefore emerge as a crisis of realizability: the reproduction-preserving systemic interest may persist while the feasible set of reproduction-preserving configurations contracts, and collapse occurs when the realized configuration can no longer maintain the reproduction threshold.

Taken as a whole, the article functions as a formalized illustration and working map of my theory of the nomenklatura and the party-state. It brings into a single mechanism the assumptions and regularities underlying my arguments about proletarian sovereignty, the command function of the nomenklatura, developmental alignment, catching-up development, discipline, systemic loyalty, crisis, and collapse. The formal structure makes it possible to recover the complete causal chain behind any particular proposition, identify the assumptions on which that proposition rests, and trace its connection to the rest of the theory. It also provides a stable framework for subsequent refinement:

additional assumptions, revised mechanisms, and new historical applications can be located within an already explicit structure. The article thus provides a detailed formal representation of the logic I invoke when describing the joint reproduction of the nomenklatura, systemic development, discipline, and the party-state.

2 The Institutional Architecture and Systemic Framework of Nomenklatura Reproduction

2.1 Constituted Second-Type Sovereignty

The model analyzes only an already constituted party-state system of second-type sovereignty. It begins after the party-state has become a working institutional form and after the nomenklatura already exists as a reproduced command body.

Formally, the model takes as given that, at time t,

$$\mathcal{N}_t \neq \emptyset,$$

so that nomenklatura positions are already reproduced through the party-state, and the party-state already functions as the institutional form through which systemic growth, discipline, and reproduction are organized.

Second-type sovereignty here means absolute proletarian sovereign power, written in the notation of *The Measure of Power* as

$$V = (0, 2).$$

Only systems satisfying this condition fall within the model's domain. Systems with relative proletarian sovereignty, non-sovereign proletarian power, bourgeois sovereignty, mixed administrative domination, or an externally reproduced dominant class do not instantiate the object of the model. They may contain bureaucracy, hierarchy, elite circulation, and administrative discipline, but they do not satisfy the condition of constituted second-type sovereignty.

Within this domain, the model asks how an already constituted party-state reproduces itself under different developmental conditions. When calibrated development remains attainable, the system can sustain a positive-development reproductive-growth regime. A realized positive-development reproductive-growth configuration requires an admissible configuration (X_t, P_t) satisfying both systemic reproduction, $S_{t+1}(X_t, P_t) \geq S_c$, and actual positive calibrated growth, $g_t(X_t, P_t) > 0$. This requirement is strictly stronger than the regime attainability condition $g_t^A > 0$. When calibrated development is no longer attainable, the system enters the exhausted-development regime and bifurcates between crisis containment and collapse.

2.2 Core Definitions

2.2.1 The Party-State and the Nomenklatura

Definition 2.1 (Party-State)**.** The *party-state* is the fused political-administrative-economic form through which the nomenklatura commands the system, organizes discipline, and directs systemic reproduction.

The party-state is the institutional form through which the nomenklatura exists as a command class-representative.

Outside the narrower terminology of the present model, this object may also be called socialism, real socialism, the Revolutionary Dictatorship of the Proletariat, or the Anticenter, depending on the level of abstraction. In the present model, however, the term party-state designates only its institutional form. The term does not replace the broader names of the formation. It marks the specific level at which the model analyzes nomenklatura reproduction.

Definition 2.2 (Nomenklatura)**.** The *nomenklatura* is the party-state's command class-representative: the fused political-managerial class of office-holders whose positions give them authority over systemic growth, discipline, and reproduction.

Formally, let

$$\mathcal{N}_t = \{1, \dots, n_t\}, \qquad n_t < \infty,$$

denote the finite set of agents who hold command positions in the party-state at time t. Thus,

$$i \in \mathcal{N}_t$$

means that agent i belongs to the nomenklatura at time t.

Definition 2.3 (Nomenklatura Agent)**.** A *nomenklatura agent* is an individual office-holder within the nomenklatura whose actions enter the reproduction of the party-state.

The model therefore works with agents

$$i \in \mathcal{N}_t$$

.

2.2.2 Hierarchy

Definition 2.4 (Hierarchical Position)**.** *Hierarchical position* is the location of a nomenklatura agent within the command hierarchy of the party-state.

Formally,

$$h_i$$

denotes the hierarchical position of agent i.

The hierarchical position is represented by a scalar index

$$h_i \in \mathcal{H} \subseteq \mathbb{R},$$

where a larger value of h_i denotes a higher position in the party-state hierarchy.

Definition 2.5 (Hierarchical Weight)**.** *Hierarchical weight* is the systemic weight attached to a nomenklatura agent's hierarchical position.

Formally,

$$w(h_i) > 0$$

denotes the weight of agent i's hierarchical position.

The weight function is differentiable in hierarchical position and satisfies

$$\frac{\partial w(h_i)}{\partial h_i} > 0.$$

Thus, higher-positioned agents enter aggregate systemic outcomes with greater weight.

2.2.3 Developedness and Efficiency Norms

Definition 2.6 (Systemic Developedness)**.** *Systemic developedness* is the internal level of productive, technological, organizational, and administrative capacity reached by the party-state system at time t.

Formally,

$$D_t > 0$$

denotes systemic developedness at time t.

Definition 2.7 (Advanced World Developedness Frontier)**.** The *advanced world developedness frontier* is the highest historically relevant level of productive, technological, organizational, and administrative developedness attained by the advanced world system at time t.

Formally,

$$D_t^w > 0$$

denotes the advanced world developedness frontier at time t.

The superscript w is retained for notational continuity with the rest of the model. The variable

$$D_t^w$$

does not represent the average level of developedness of all countries or of the entire world economy. It represents the external developmental frontier set by the most advanced part of the world system against which the party-state's own developedness is measured.

The party-state analyzed by the model is a catching-up system relative to this advanced external frontier. The active developmental domain of the present model is therefore strictly pre-frontier:

$$\boxed{0 < D_t < D_t^w.}$$

Thus a developmental gap remains throughout every period whose transition dynamics are analyzed in this paper. The boundary condition

$$D_t = D_t^w$$

represents attainment of the advanced world developedness frontier and lies outside the active transition domain of the present model. No post-attainment dynamics are specified here.

Definition 2.8 (Relative Developedness)**.** *Relative developedness* is the developedness of the party-state measured as a proportion of the advanced world developedness frontier.

Formally,

$$\rho_t = \frac{D_t}{D_t^w}.$$

Since the active developmental domain satisfies

$$0 < D_t < D_t^w,$$

relative developedness satisfies

$$\boxed{0 < \rho_t < 1.}$$

An increase in

$$\rho_t$$

means that the party-state closes part of its developmental gap relative to the advanced world frontier.

A constant

$$\rho_t$$

means that the party-state maintains the same proportional developmental position relative to that frontier.

A decrease in

$$\rho_t$$

means that the party-state falls behind the advancing world frontier in relative terms.

The boundary value

$$\rho_t = 1$$

represents equality with the advanced world developedness frontier. It is not an interior state of the present model. The transition dynamics analyzed here apply only while

$$0 < \rho_t < 1.$$

Accordingly, movements in

$$\rho_t$$

are interpreted as movements in the party-state's relative developmental distance from the historically leading external level of developedness while a positive developmental gap remains.

Definition 2.9 (Historical Efficiency Norm)**.** The *historical efficiency norm* is the historically realized rate at which systemic developedness increases between two periods.

Formally,

$$g_t = \frac{D_{t+1} - D_t}{D_t}.$$

Equivalently,

$$D_{t+1} = D_t(1 + g_t).$$

Definition 2.10 (Advanced World Frontier Growth Rate)**.** The *advanced world frontier growth rate* is the rate at which the advanced world developedness frontier increases between periods t and $t + 1$.

Formally,

$$g_t^w = \frac{D_{t+1}^w - D_t^w}{D_t^w}.$$

Equivalently,

$$D_{t+1}^{w} = D_t^{w}(1 + g_t^{w}).$$

Assumption 2.11 (Advancing World Developedness Frontier)**.** Over the historical domain analyzed by the model, the advanced world developedness frontier continues to advance:

$$g_t^{w} > 0.$$

Hence,

$$D_{t+1}^{w} > D_t^{w}.$$

The external frontier growth rate

$$g_t^{w}$$

must be distinguished from the party-state's frontier efficiency norm

$$g_t^{\max}.$$

The variable

$$g_t^{w}$$

describes the realized movement of the external advanced world developedness frontier. By contrast,

$$g_t^{\max}$$

denotes the maximum rate of increase in the party-state's own systemic developedness attainable under the historical conditions of period t.

Interior transition domain. Because the present model analyzes only pre-frontier dynamics, a t-to-$t+1$ transition belongs to the active modeled domain only when both the inherited and resulting relative-developedness states remain strictly interior:

$$0 < \rho_t < 1, \qquad 0 < \rho_{t+1} < 1.$$

Using the law of motion derived below, the upper-domain condition for the resulting state is equivalently

$$\rho_t \frac{1 + g_t}{1 + g_t^{w}} < 1.$$

Hence the model studies developmental transitions only while the party-state remains strictly below the advanced world developedness frontier. A transition reaching the frontier boundary terminates the developmental domain analyzed in this paper rather than initiating a post-frontier regime within the present model.

Proposition 2.12 (Law of Motion of Relative Developedness)**.** *For every t-to-$t+1$ transition within the active interior developmental domain, relative developedness evolves according to*

$$\boxed{\rho_{t+1} = \rho_t \frac{1 + g_t}{1 + g_t^{w}}.}$$

Consequently,

$$g_t > g_t^w \iff \rho_{t+1} > \rho_t,$$

$$g_t = g_t^w \iff \rho_{t+1} = \rho_t,$$

and

$$g_t < g_t^w \iff \rho_{t+1} < \rho_t.$$

Proof. By definition,

$$\rho_{t+1} = \frac{D_{t+1}}{D_{t+1}^w}.$$

Using

$$D_{t+1} = D_t(1 + g_t)$$

and

$$D_{t+1}^w = D_t^w(1 + g_t^w),$$

obtain

$$\rho_{t+1} = \frac{D_t(1 + g_t)}{D_t^w(1 + g_t^w)} = \rho_t \frac{1 + g_t}{1 + g_t^w}.$$

Since

$$\rho_t > 0$$

and

$$1 + g_t^w > 0,$$

the direction of change in relative developedness is determined by the comparison between

$$g_t$$

and

$$g_t^w.$$

Therefore,

$$\rho_{t+1} > \rho_t$$

if and only if

$$g_t > g_t^w,$$

with equality and the reverse inequality following analogously. □

The comparison between

$$g_t$$

and

$$g_t^w$$

therefore determines movement within the pre-frontier developmental domain. The present model does not extend this transition system beyond

$$\rho = 1.$$

The proposition is an accounting relation and does not by itself define a development or crisis regime. In particular, positive absolute growth,

$$g_t > 0,$$

does not necessarily imply increasing relative developedness. If

$$0 < g_t < g_t^w,$$

then the party-state continues to increase its absolute systemic developedness,

$$D_{t+1} > D_t,$$

while simultaneously losing ground relative to the advancing world frontier,

$$\rho_{t+1} < \rho_t.$$

The distinction between absolute positive development, catching-up development, and relative developmental erosion will be used explicitly in the formal results below.

Definition 2.13 (Frontier Efficiency Norm)**.** The *frontier efficiency norm* is the maximum attainable rate of increase in systemic developedness under the historical conditions of period *t*.

Formally,

$$g_t^{\max}$$

denotes the frontier efficiency norm at time *t*.

The historical efficiency norm cannot exceed the frontier efficiency norm:

$$g_t \leq g_t^{\max}.$$

Because $D_t > 0$ and $D_{t+1} > 0$, the definition implies $g_t > -1$. No additional lower bound beyond this definitional restriction is imposed on g_t. In the positive-development regime analyzed below, the realized calibrated historical efficiency norm is positive, whereas in the exhausted-development or crisis regime, the historical efficiency norm may be zero or negative.

2.2.4 Nomenklatura Actions

Definition 2.14 (Developmental Effort)**.** *Developmental effort* is a nomenklatura agent's contribution to increasing the systemic developedness of the party-state.

Formally,

$$x_{i,t} \geq 0$$

denotes the developmental effort of agent *i* at time *t*.

Definition 2.15 (Voice)**.** *Voice* is a nomenklatura agent's revealed position toward discipline within the party-state.

Formally,

$$v_{i,t}$$

denotes the realized voice position of agent i in force at time t.

The term *voice* refers to revealed support for, or resistance to, the disciplinary level of the party-state. The period-t voice position is part of the realized state inherited at the beginning of the period. During period t, the agent may adjust that position, producing the next-period voice position

$$v_{i,t+1}.$$

Definition 2.16 (Discipline)**.** *Discipline* is the realized party-state regime of coercive and administrative control that constrains deviation and enforces systemic reproduction.

Formally,

$$P_t \geq 0$$

denotes the realized level of discipline in force at time t.

Discipline includes administrative control, surveillance, cadre discipline, sanction, and punishment. It is therefore broader than punishment alone. The level P_t is part of the realized period-t state under which developmental effort and other period-t actions take place.

2.2.5 Timing Convention and One-Period Transition

The model uses a one-period transition convention. At the beginning of period t, the party-state and its nomenklatura inherit a realized systemic state characterized by

$$D_t, \qquad \rho_t, \qquad S_t, \qquad P_t, \qquad (v_{i,t})_{i \in \mathcal{N}_t}.$$

Aggregate voice in force at that state is

$$V_t = \sum_{i \in \mathcal{N}_t} w(h_i) v_{i,t}.$$

The systemic utility

$$U_{i,t}^{sys}$$

is evaluated at this realized period-t state. It records the systemic class benefit, residual deviation rent, and disciplinary exposure available under the developedness, stability, and disciplinary conditions already in force at time t.

During period t, nomenklatura agents supply developmental effort

$$x_{i,t}$$

and may adjust their inherited voice positions from

$$v_{i,t}$$

to

$$v_{i,t+1}.$$

Aggregate developmental effort

$$X_t = \sum_{i \in \mathcal{N}_t} w(h_i) x_{i,t}$$

operates under the realized disciplinary environment

$$P_t$$

and determines the historical efficiency norm

$$g_t(X_t, P_t).$$

The resulting one-period developmental transition is

$$D_{t+1} = D_t\big[1 + g_t(X_t, P_t)\big],$$

$$\rho_{t+1} = \rho_t \frac{1 + g_t(X_t, P_t)}{1 + g_t^w},$$

and

$$S_{t+1} = H(\rho_{t+1}, P_t).$$

The period-t voice adjustment produces

$$V_{t+1} = \sum_{i \in \mathcal{N}_{t+1}} w(h_i) v_{i,t+1},$$

which enters the next-period disciplinary state. Consistently with the general voice-to-discipline mapping,

$$P^{\Phi}_{t+1}(V_{t+1}) = \Phi(V_{t+1}, D_{t+1}, \rho_{t+1}),$$

and realized next-period discipline is bounded by next-period disciplinary capacity:

$$P_{t+1} = P^{RZ}_{t+1}(V_{t+1}) = \min\big\{P^{\Phi}_{t+1}(V_{t+1}), \bar{P}_{t+1}\big\}.$$

Thus the period-t state is inherited rather than contemporaneously produced by period-t action. Period-t effort and voice adjustment instead contribute to the state realized in period $t+1$. In compact form,

$$\boxed{\text{realized state at } t \;\longrightarrow\; \text{period-}t \text{ actions} \;\longrightarrow\; \text{realized state at } t+1.}$$

Unless a later result explicitly models hierarchical reassignment, the hierarchical position h_i is treated as fixed over a single t-to-$t+1$ transition.

2.2.6 Systemic Aggregates

Definition 2.17 (Aggregate Developmental Effort)**.** *Aggregate developmental effort* is the hierarchical-weighted sum of individual developmental efforts within the nomenklatura.

Formally,

$$X_t = \sum_{i \in \mathcal{N}_t} w(h_i) x_{i,t}.$$

Aggregate developmental effort links nomenklatura action to systemic developedness through the historical efficiency norm:

$$X_t \longrightarrow g_t \longrightarrow D_{t+1}.$$

The function

$$\Gamma(X_t, g_t^{\max})$$

specifies the frontier-directed component of the historical efficiency norm generated by aggregate developmental effort. It satisfies

$$0 \le \Gamma(X_t, g_t^{\max}) \le g_t^{\max},$$

$$\frac{\partial \Gamma}{\partial X_t} > 0,$$

and

$$\frac{\partial \Gamma}{\partial g_t^{\max}} > 0.$$

The calibrated historical efficiency norm, which subtracts harm from deviation and the cost of discipline, is defined later in the propositions section.

Definition 2.18 (Aggregate Voice)**.** *Aggregate voice* is the hierarchical-weighted sum of nomenklatura agents' revealed positions toward discipline.

Formally,

$$V_t = \sum_{i \in \mathcal{N}_t} w(h_i) v_{i,t}.$$

Aggregate voice links the realized voice profile in force at time t to the realized disciplinary level of the same period:

$$V_t \longrightarrow P_t.$$

The mapping

$$P_t^{\Phi}(V_t) = \Phi(V_t, D_t, \rho_t)$$

defines the raw voice-induced disciplinary level associated with the realized period-t state. Under the timing convention above, this relation is a state identity: the period-t pair (V_t, P_t) has already been realized when period-t actions are taken. Voice adjustment during period t produces V_{t+1}, which enters the corresponding next-period mapping for P_{t+1}. The actually realizable disciplinary level is defined below once disciplinary capacity is introduced.

2.2.7 Systemic Stability

Definition 2.19 (Systemic Stability)**.** *Systemic stability* is the degree to which the party-state remains capable of reproducing itself as an institutional form.

Formally,

$$S_t \in [0, 1]$$

denotes systemic stability at time t.

The system reproduces when stability remains above the critical reproduction threshold:

$$S_t \ge S_c,$$

where

$$S_c \in (0, 1).$$

When

$$S_t < S_c,$$

the systemic reproduction threshold is violated. This condition represents reproduction failure at the realized state; it does not by itself define the exhausted-development crisis regime, which is introduced separately through the attainable calibrated developmental frontier. Systemic stability depends on relative developedness and discipline:

$$S_{t+1} = H(\rho_{t+1}, P_t),$$

with

$$\frac{\partial H}{\partial \rho_{t+1}} > 0$$

and

$$\frac{\partial H}{\partial P_t} > 0.$$

Definition 2.20 (Disciplinary Capacity)**.** *Disciplinary capacity* is the maximum level of discipline that the party-state can realize under its available developedness, relative developedness, and stability conditions. It is denoted by

$$\bar{P}_t = \bar{P}(D_t, \rho_t, S_t),$$

with

$$\frac{\partial \bar{P}}{\partial D_t} > 0, \qquad \frac{\partial \bar{P}}{\partial \rho_t} > 0, \qquad \frac{\partial \bar{P}}{\partial S_t} > 0.$$

The actually realized discipline is therefore bounded by capacity:

$$P_t^{RZ}(V_t) = \min\left\{P_t^{\Phi}(V_t), \bar{P}_t\right\}.$$

Thus, for the realized period-t state,

$$P_t = P_t^{RZ}(V_t).$$

The same state identity applies one period forward after the period-t voice adjustment and systemic transition have generated V_{t+1}, D_{t+1}, ρ_{t+1}, S_{t+1}, and $\bar{P}_{t+1}$.

Disciplinary capacity is a general constraint of the model. It need not bind in the positive-development regime, but it may become binding under exhausted development.

2.2.8 Systemic Class Benefit

Definition 2.21 (Systemic Class Benefit)**.** *Systemic class benefit* is the benefit a nomenklatura agent receives from occupying a hierarchical position within a developed party-state system.

Formally,

$$Z(D_t, h_i) > 0$$

denotes the systemic class benefit of agent i at time t, where D_t is systemic developedness and h_i is the agent's hierarchical position.

The function $Z(D_t, h_i)$ is differentiable in both arguments on its domain.

The systemic class benefit is assumed to increase with systemic developedness:

$$\frac{\partial Z(D_t, h_i)}{\partial D_t} > 0,$$

and with hierarchical position:

$$\frac{\partial Z(D_t, h_i)}{\partial h_i} > 0.$$

This means that the benefit of a nomenklatura position depends both on the developedness of the party-state system and on the location of the agent within the command hierarchy.

The stability-adjusted systemic class benefit is

$$S_t Z(D_t, h_i).$$

This term captures the fact that a position within the party-state yields its full systemic class benefit only insofar as the party-state remains stable enough to reproduce itself.

2.2.9 Risk, Deviation, and Collapse-Contingent Market Exposure

Definition 2.22 (Deviation)**.** *Deviation* is a nomenklatura agent's private use of systemic position for off-system gain in a way that may or may not impose harm on systemic reproduction.

Formally,

$$e_{i,t} \geq 0$$

denotes the deviation of agent i at time t.

Let

$$b_i(e_i, D_t, h_i)$$

denote the private benefit from deviation e_i, and let

$$f_i(e_i, h_i)$$

denote the penalty or loss imposed if deviation is detected and sanctioned.

The private benefit from deviation and the sanction loss are nonnegative:

$$b_i(e_i, D_t, h_i) \geq 0, \qquad f_i(e_i, h_i) \geq 0$$

for every admissible $e_i > 0$.

Let

$$q_i(P_t, h_i) \in [0, 1]$$

denote the probability that deviation is detected and sanctioned under discipline P_t. This probability is assumed to increase with discipline:

$$\frac{\partial q_i(P_t, h_i)}{\partial P_t} > 0.$$

The expected private return from deviation is

$$\mathbb{E}[R_i(e_i, P_t, D_t, h_i)] = \big(1 - q_i(P_t, h_i)\big)b_i(e_i, D_t, h_i) - q_i(P_t, h_i)f_i(e_i, h_i).$$

Definition 2.23 (Residual Deviation Rent)**.** *Residual deviation rent* is the nonnegative expected private gain from deviation that remains profitable under a given level of discipline.

Formally,

$$r_i(P_t, D_t, h_i) = \max\left\{0, \sup_{e_i>0} \mathbb{E}[R_i(e_i, P_t, D_t, h_i)]\right\}.$$

Equivalently,

$$r_i(P_t, D_t, h_i) = \max\left\{0, \sup_{e_i>0} \left[\big(1 - q_i(P_t, h_i)\big)b_i(e_i, D_t, h_i) - q_i(P_t, h_i)f_i(e_i, h_i)\right]\right\}.$$

If

$$\mathbb{E}[R_i(e_i, P_t, D_t, h_i)] \le 0$$

for all $e_i > 0$, then

$$r_i(P_t, D_t, h_i) = 0.$$

If there exists some $e_i > 0$ such that

$$\mathbb{E}[R_i(e_i, P_t, D_t, h_i)] > 0,$$

then residual deviation rent is positive:

$$r_i(P_t, D_t, h_i) > 0.$$

Definition 2.24 (Erroneous or Excessive Disciplinary Exposure)**.** *Erroneous or excessive disciplinary exposure* is the probability that a nomenklatura agent is subjected to mistaken, excessive, or politically misdirected discipline under the party-state's disciplinary regime.

Formally,

$$m_i(P_t, h_i) \in [0, 1]$$

denotes the disciplinary exposure of agent *i* at time *t*.

The term $m_i(P_t, h_i)$ refers to mistaken, excessive, or politically misdirected punishment that may occur under the disciplinary apparatus itself.

Definition 2.25 (Disciplinary Loss)**.** *Disciplinary loss* is the loss incurred by a nomenklatura agent if mistaken or excessive discipline falls on that agent.

Formally,

$$F_i(h_i) > 0$$

denotes the disciplinary loss of agent *i*.

The expected loss from disciplinary exposure is therefore

$$m_i(P_t, h_i)F_i(h_i).$$

Definition 2.26 (All-Cause Disciplinary Displacement)**.** Let

$$d_{i,t}^{D} = d_i^{D}(P_t, h_i) \in [0, 1]$$

denote the reduced-form probability that agent i's currently reproduced nomenklatura position is terminated or otherwise ceases to be held by that agent during the relevant period as a result of serious party-state disciplinary action, regardless of whether that action is justified, deviation-related, mistaken, excessive, administratively motivated, or politically directed.

The object

$$d_{i,t}^{D}$$

is an all-cause incidence measure. It is not an additional payoff loss and is not added to systemic utility.

The all-cause displacement probability must be distinguished from both

$$q_i(P_t, h_i)$$

and

$$m_i(P_t, h_i).$$

The first is a sanction probability conditional on deviation; the second is the erroneous-or-excessive disciplinary-loss channel already entering systemic utility. By contrast,

$$d_{i,t}^{D}$$

is an unconditional reduced-form measure of serious individual displacement from a currently reproduced nomenklatura position across all disciplinary causes.

Accordingly,

$$d_{i,t}^{D} \neq q_i(P_t, h_i) + m_i(P_t, h_i)$$

in general, and the model does not construct it by summing those two probabilities.

Let the expected headcount incidence of all-cause disciplinary displacement be

$$\bar{d}_t^{D,N} = \frac{1}{n_t} \sum_{i \in \mathcal{N}_t} d_{i,t}^{D}.$$

Let the corresponding hierarchical-weighted incidence be

$$\bar{d}_t^{D,w} = \frac{\sum_{i \in \mathcal{N}_t} w(h_i) d_{i,t}^{D}}{\sum_{i \in \mathcal{N}_t} w(h_i)}.$$

Assumption 2.27 (Sparse All-Cause Disciplinary Displacement under Systemic Reproduction)**.** Within the reproduced-system domain in which the nomenklatura continues to function as the party-state's command body, all-cause position-destroying disciplinary displacement remains structurally sparse.

There exist parameters

$$\varepsilon_D^N > 0, \qquad \varepsilon_D^w > 0,$$

maintained as small relative to unity, such that

$$\boxed{\bar{d}_t^{D,N} \leq \varepsilon_D^N \ll 1}$$

and

$$\boxed{\bar{d}_t^{D,w} \leq \varepsilon_D^w \ll 1.}$$

No specific numerical calibration is imposed.

This is an institutional reproduction condition rather than a claim that discipline is absent. A functioning party-state may sanction, remove, demote, punish, or replace particular nomenklatura agents. What it cannot do as its normal reproduction mechanism is destroy a non-small share of the command body through which systemic administration and reproduction are themselves organized.

Individual displacement therefore occurs inside a reproduced institutional structure. The nomenklatura position as an institutional form continues to exist and can continue to be occupied and reproduced even when particular incumbents are removed.

Definition 2.28 (Collapse-Contingent Non-Reproduction of the Nomenklatura Position)**.** For an agent

$$i \in \mathcal{N}_t,$$

let

$$\ell_{i,t+1}^C$$

denote the indicator that the agent's period-t nomenklatura position is not reproduced as a nomenklatura position in period $t+1$ because the party-state enters the collapse branch.

If

$$S_{t+1} < S_c,$$

then by definition of systemic reproduction failure,

$$\boxed{\ell_{i,t+1}^C = 1 \qquad \text{for every } i \in \mathcal{N}_t.}$$

This statement concerns the institutional position, not the physical survival of the person. Former nomenklatura agents may remain alive, retain assets, enter private employment, become entrepreneurs, or obtain other post-collapse positions. What is no longer reproduced is their status and position as members of the party-state nomenklatura.

The headcount incidence of non-reproduction of inherited nomenklatura positions under collapse is therefore

$$\bar{\ell}_{t+1}^{C,N} = \frac{1}{n_t} \sum_{i \in \mathcal{N}_t} \ell_{i,t+1}^C = 1.$$

Its hierarchical-weighted counterpart is likewise

$$\bar{\ell}_{t+1}^{C,w} = \frac{\sum_{i \in \mathcal{N}_t} w(h_i) \ell_{i,t+1}^C}{\sum_{i \in \mathcal{N}_t} w(h_i)} = 1.$$

Proposition 2.29 (Structural Asymmetry of Disciplinary Displacement and Collapse)**.** *Under Assumption 2.27, all-cause serious disciplinary displacement inside a reproduced party-state affects only a structurally small share of the nomenklatura:*

$$\bar{d}_t^{D,N} \leq \varepsilon_D^N \ll 1,$$

and

$$\bar{d}_t^{D,w} \leq \varepsilon_D^w \ll 1.$$

By contrast, conditional on systemic collapse,

$$S_{t+1} < S_c,$$

the inherited nomenklatura position is not reproduced for the entire period-t nomenklatura:

$$\bar{\ell}_{t+1}^{C,N} = \bar{\ell}_{t+1}^{C,w} = 1.$$

Hence the institutional incidence gap satisfies

$$\boxed{\bar{\ell}_{t+1}^{C,N} - \bar{d}_t^{D,N} \geq 1 - \varepsilon_D^N}$$

and

$$\boxed{\bar{\ell}_{t+1}^{C,w} - \bar{d}_t^{D,w} \geq 1 - \varepsilon_D^w.}$$

Proof. The disciplinary bounds follow directly from Assumption 2.27. Under collapse,

$$S_{t+1} < S_c,$$

the party-state fails to reproduce as the institutional form through which nomenklatura positions exist. Therefore

$$\ell_{i,t+1}^C = 1$$

for every inherited nomenklatura agent, so both the headcount and hierarchical-weighted non-reproduction incidences equal unity. Subtracting the reproduced-system disciplinary-incidence bounds yields the stated inequalities. ☐

Structural asymmetry of disciplinary and collapse exposure. The two downside structures are not symmetric risks of the same event.

Inside a reproduced party-state, particular nomenklatura agents may face disciplinary loss. The utility-level erroneous-or-excessive loss channel remains

$$m_i(P_t, h_i)F_i(h_i),$$

while the broader all-cause incidence of serious individual displacement is summarized by

$$d_{i,t}^D.$$

By Assumption 2.27, such displacement remains sparse relative to the size and hierarchical weight of the reproduced nomenklatura body.

Collapse is categorically different. If

$$S_{t+1} < S_c,$$

the nomenklatura position itself ceases to be reproduced as an institutional position of the party-state. Thus the class-wide incidence of loss of the inherited nomenklatura position equals unity:

$$\bar{\ell}^{C,N}_{t+1} = \bar{\ell}^{C,w}_{t+1} = 1.$$

Only after this institutional loss has occurred does the separate market-conversion lottery become relevant:

$$O_i^C = \pi_i^C M_i^C - (1 - \pi_i^C) L_i^C.$$

Accordingly,

$$m_i(P_t, h_i)$$

and

$$1 - \pi_i^C$$

must not be interpreted as two directly comparable probabilities of losing the same position. They condition different events. The first belongs to an intra-system disciplinary-loss channel; the second is the probability of unsuccessful market conversion after the nomenklatura position has already ceased to be reproduced.

The relevant institutional comparison is therefore:

$$\boxed{\text{sparse individual displacement inside a reproduced command body} \quad \text{versus} \quad \text{universal non-reproduction of the inh}}$$

Definition 2.30 (Collapse-Contingent Market Exposure)**.** *Collapse-contingent market exposure* is the risk-adjusted expected payoff faced by a nomenklatura agent if the party-state fails to reproduce, the agent's protected party-state position is destroyed, and the agent becomes exposed to post-collapse market conversion.

Formally,

$$O_i^C$$

denotes the collapse-contingent market exposure of agent *i*.

It is the payoff condition that becomes relevant if party-state reproduction fails; it is not a payoff from voluntary resignation from the party-state. Once the nomenklatura position ceases to be reproduced, the agent is exposed to post-collapse market conversion. The outside condition is therefore not a risk-free benchmark: successful conversion yields

$$M_i^C,$$

whereas unsuccessful or adverse conversion generates the downside

$$L_i^C.$$

The collapse-contingent market exposure may be written as

$$O_i^C = \pi_i^C M_i^C - (1 - \pi_i^C) L_i^C,$$

The probability

$$\pi_i^C$$

is not the probability that the agent retains the nomenklatura position after collapse. Conditional on collapse, that inherited party-state position is not reproduced. Instead,

$$\pi_i^C$$

is the probability of successful conversion into a different post-collapse market position after the nomenklatura position has already been lost as a systemic position.

Where

$$\pi_i^C \in [0, 1]$$

is the probability of successful post-collapse market conversion,

$$M_i^C$$

is the payoff from successful post-collapse conversion, and

$$L_i^C$$

is the downside loss associated with unsuccessful or adverse post-collapse market conversion. This loss may include failure to convert a party-state position into a protected market position, loss of status or income, loss or devaluation of assets, competitive displacement, or other adverse realizations associated with exposure to the post-collapse market environment.

2.2.10 Systemic Utility of the Nomenklatura Agent

Definition 2.31 (Systemic Utility)**.** *Systemic utility* is the payoff the agent receives from remaining within and reproducing the party-state system.

Formally,

$$U_{i,t}^{sys} = S_t Z(D_t, h_i) + r_i(P_t, D_t, h_i) - m_i(P_t, h_i) F_i(h_i).$$

The first term,

$$S_t Z(D_t, h_i),$$

is the stability-adjusted systemic class benefit.

The second term,

$$r_i(P_t, D_t, h_i),$$

is residual deviation rent.

The third term,

$$m_i(P_t, h_i) F_i(h_i),$$

is expected disciplinary loss.

Thus, the nomenklatura agent's systemic utility combines the benefit of position inside the party-state, any residual private rent that remains possible under discipline, and the expected loss generated by disciplinary exposure.

The negative disciplinary-exposure term is therefore already internalized in systemic utility. The model does not compare a risk-adjusted party-state position with a risk-free market alternative. Rather, it compares two complete payoff conditions:

$$\underbrace{S_t Z(D_t, h_i) + r_i(P_t, D_t, h_i) - m_i(P_t, h_i) F_i(h_i)}_{\text{systemic position with disciplinary exposure}}$$

and

$$\underbrace{\pi_i^C M_i^C - (1 - \pi_i^C) L_i^C}_{\text{collapse-contingent market exposure}} \quad .$$

The payoff comparison remains necessary because individual payoff magnitudes are heterogeneous. Proposition 2.29, however, establishes that the institutional exposure underlying the two branches is strongly asymmetric.

Inside the reproduced system, serious all-cause individual displacement is sparse at the class level. Under collapse, loss of the inherited nomenklatura position is universal by construction. Post-collapse market conversion is therefore a secondary individual lottery following the prior destruction of the reproduced party-state position.

The quantity

$$U_{i,t}^{sys}$$

is evaluated at the realized period-t state

$$(D_t, \rho_t, S_t, P_t).$$

It is therefore the systemic payoff available under conditions already in force at time t. Period-t developmental effort and voice adjustment affect the subsequent systemic state and hence the systemic utility realized in period $t + 1$; they do not retroactively determine the already realized value of

$$U_{i,t}^{sys}.$$

Definition 2.32 (Systemic Loyalty Condition)**.** A nomenklatura agent remains *system-loyal* when systemic utility inside a reproduced party-state is at least as large as collapse-contingent market exposure.

Definition 2.33 (Decisively Weighted Body)**.** A subset

$$B_t \subseteq \mathcal{N}_t$$

is *decisively weighted for systemic reproduction* at a realized reproduction-feasible configuration if a coordinated withdrawal of reproduction-preserving developmental effort and/or discipline-calibrating voice by the agents in B_t, holding the actions of agents in

$$\mathcal{N}_t \setminus B_t$$

fixed, is sufficient to induce an effort-discipline configuration

$$(\widehat{X}_t, \hat{P}_t)$$

for which

$$S_{t+1}(\widehat{X}_t, \hat{P}_t) < S_c.$$

No exogenous weighted-share threshold is imposed. The term is defined by the systemic consequence of the coordinated withdrawal.

The definition is counterfactual. It identifies a body whose coordinated withdrawal would be sufficient to violate systemic reproduction if that withdrawal were undertaken. It does not imply that the body is already organized as a coalition, that coordination occurs automatically, or that a shared preference for alternative conduct is sufficient by itself to produce collapse.

Realized-state loyalty and continuation loyalty. The systemic loyalty comparison at time t,

$$U_{i,t}^{sys} \geq O_i^C,$$

is a realized-state comparison. It describes the agent's payoff position under the systemic conditions already in force at time t. Under the timing convention of Subsection 2.2.5, it is not the payoff generated by period-t developmental effort or period-t voice adjustment.

When the model evaluates a period-t action whose consequences determine whether the party-state is reproduced in period $t+1$, the relevant inside-system payoff is instead the systemic utility of the resulting reproduced continuation state.

Formally, agent i remains system-loyal at time t if

$$U_{i,t}^{sys} \geq O_i^C.$$

That is,

$$S_t Z(D_t, h_i) + r_i(P_t, D_t, h_i) - m_i(P_t, h_i) F_i(h_i) \geq O_i^C.$$

This condition compares two risk-adjusted institutional positions. The systemic side includes the benefits of a reproduced nomenklatura position, residual private rent, and expected disciplinary downside. The collapse-contingent side includes the possibility of successful market conversion and the downside associated with unsuccessful or adverse market exposure after party-state reproduction fails.

Consequently,

$$m_i(P_t, h_i) F_i(h_i) > 0$$

does not imply

$$O_i^C > U_{i,t}^{sys}.$$

The existence of disciplinary exposure inside the party-state is therefore not sufficient to establish that collapse-contingent market conversion is preferable.

When this condition holds for the reproduction-preserving body of the nomenklatura, reproduction-preserving conduct remains individually rational for those agents.

When the inequality is reversed,

$$U_{i,t}^{sys} < O_i^C,$$

collapse-contingent market exposure dominates systemic reproduction for agent i.

Definition 2.34 (One-Period Systemic Continuation Payoff)**.** Let

$$a_t$$

denote a contemplated period-t action profile, and let

$$\Omega_{t+1}(a_t) = (D_{t+1}(a_t), \rho_{t+1}(a_t), S_{t+1}(a_t), P_{t+1}(a_t))$$

denote the resulting period-$t+1$ systemic state generated by the transition structure of the model.

If the contemplated action profile preserves party-state reproduction,

$$S_{t+1}(a_t) \geq S_c,$$

define the one-period systemic continuation payoff of agent i by

$$\mathscr{C}_{i,t}^{sys}(a_t) = S_{t+1}(a_t)Z(D_{t+1}(a_t), h_i) + r_i\left(P_{t+1}(a_t), D_{t+1}(a_t), h_i\right) - m_i\left(P_{t+1}(a_t), h_i\right) F_i(h_i).$$

Equivalently,

$$\mathscr{C}_{i,t}^{sys}(a_t) = U_{i,t+1}^{sys}\big(\Omega_{t+1}(a_t)\big)$$

on the reproduction-preserving branch.

If instead the contemplated period-t action profile violates the reproduction threshold,

$$S_{t+1}(a_t) < S_c,$$

the nomenklatura position is not reproduced in the resulting state. The systemic continuation payoff

$$\mathscr{C}_{i,t}^{sys}(a_t)$$

is therefore not the realized payoff on that branch. The relevant payoff condition becomes collapse-contingent market exposure,

$$O_i^C,$$

or, when current disciplinary blocking is analytically relevant,

$$\widetilde{O}_i^C(P_t).$$

Thus the period-t reproduction choice compares a reproduced next-period systemic position with a collapse-contingent outside condition:

$$\boxed{\text{reproduction-preserving continuation payoff} \quad \text{vs} \quad \text{collapse-contingent market exposure.}}$$

Definition 2.35 (One-Period Continuation Loyalty)**.** Let

$$a_t^R$$

denote a contemplated reproduction-preserving period-t action profile satisfying

$$S_{t+1}(a_t^R) \geq S_c.$$

Agent i satisfies *one-period continuation loyalty* relative to the unblocked collapse-contingent alternative when

$$\boxed{\mathscr{C}_{i,t}^{sys}(a_t^R) \geq O_i^C.}$$

When the effective collapse-contingent alternative is evaluated under the realized period-t disciplinary environment, continuation loyalty is

$$\boxed{\mathscr{C}_{i,t}^{sys}(a_t^R) \geq \widetilde{O}_i^C(P_t).}$$

One-period continuation loyalty is the preference condition relevant to period-t conduct whose systemic consequence is realized in period $t+1$. It must be distinguished from realized-state loyalty,

$$U_{i,t}^{sys} \geq O_i^C,$$

which describes the payoff position already inherited at time t.

The model does not assume that realized-state loyalty at t mechanically implies continuation loyalty for every possible $t+1$ state. Whenever continuation loyalty is required for a behavioral result, it is stated explicitly.

3 The Logic of Nomenklatura Reproduction Under a Positive Frontier Efficiency Norm and Its Formal Results

Positive-development regime. Within the domain of already constituted second-type sovereign party-state systems, the formal results in this part of the model are stated under a positive-development regime.

This condition is stronger than the positivity of the raw developmental frontier. The inequality

$$g_t^{\max} > 0$$

only indicates that the historical frontier still permits development. It does not yet imply that the party-state can realize positive development once harm from deviation and the cost of discipline are internalized. The relevant regime condition is therefore that the attainable calibrated developmental frontier remains positive:

$$g_t^A > 0.$$

Equivalently, there exists a feasible effort-discipline configuration under which calibrated growth remains positive.

The attainable calibrated developmental frontier is formally defined below in Subsection 3.6. Under the positive-development regime, maximum accessible developmental effort and system-optimal discipline are jointly attainable, and the maintained condition

$$g_t^A > 0$$

therefore means that positive calibrated development remains available after internal deviation harm and disciplinary cost have been fully calibrated. In the present positive-development regime, the model assumes that positive calibrated development remains attainable. Under this regime, systemic developedness can still grow because positive calibrated development remains attainable, and the nomenklatura's systemic class benefit remains tied to the reproduction and development of the party-state. The formal results below describe the logic of nomenklatura reproduction while positive calibrated development remains attainable. The exhausted-development case, in which the attainable calibrated developmental frontier is non-positive, is treated separately in the crisis part of the model.

Catching-up development and relative developmental position. The positive-development regime must be distinguished from the narrower condition of catching-up development.

By Proposition 2.12, relative developedness evolves according to

$$\rho_{t+1} = \rho_t \frac{1 + g_t}{1 + g_t^w}.$$

The party-state possesses a characteristic catching-up developmental property over the historically expansive phase of its positive-development trajectory. A period is a *catching-up period* when

$$g_t > g_t^w,$$

in which case

$$\rho_{t+1} > \rho_t.$$

The party-state then closes part of the developmental gap separating it from the advanced world developedness frontier.

If

$$g_t = g_t^w,$$

then

$$\rho_{t+1} = \rho_t,$$

and the party-state preserves its existing proportional position relative to the advanced world frontier.

If instead

$$0 < g_t < g_t^w,$$

then

$$D_{t+1} > D_t$$

while

$$\rho_{t+1} < \rho_t.$$

The party-state continues to develop in absolute terms but loses ground relative to the faster-moving advanced world frontier.

These cases must not be conflated with the positive-development regime condition

$$g_t^A > 0.$$

The condition

$$g_t^A > 0$$

means that positive calibrated development remains attainable. It does not require realized growth to exceed the external frontier growth rate in every period.

Accordingly, a party-state may remain within the positive-development reproductive-growth regime even during a period of relative developmental erosion,

$$0 < g_t < g_t^w,$$

provided that positive calibrated growth remains attainable and the developmental, reproductive, disciplinary-capacity, and strategic-blocking conditions required for systemic reproduction continue to hold.

The model therefore distinguishes:

$$\boxed{\text{catching-up development} \neq \text{positive calibrated development} \neq \text{relative developmental maintenance.}}$$

Historically, sustained periods of

$$g_t > g_t^w$$

allow the party-state to accumulate relative developedness and strengthen the material, organizational, and administrative base on which its command capacity depends. A later fall of realized growth below

$$g_t^w$$

does not by itself constitute exhausted development or systemic crisis. It means only that the previously accumulated relative developmental position begins to erode.

Whether such relative erosion becomes systemically destabilizing depends on the remaining developmental and command-capacity margins of the party-state. Those margins are formalized below and become central to the exhausted-development analysis.

Definition 3.1 (Catching-Up Development)**.** The party-state is in a period of *catching-up development* when its realized calibrated historical efficiency norm exceeds the growth rate of the advanced world developedness frontier:

$$g_t > g_t^w.$$

Equivalently,

$$\rho_{t+1} > \rho_t.$$

Definition 3.2 (Relative Developmental Erosion)**.** The party-state experiences *relative developmental erosion* when its realized calibrated historical efficiency norm is lower than the growth rate of the advanced world developedness frontier:

$$g_t < g_t^w.$$

Equivalently,

$$\rho_{t+1} < \rho_t.$$

Relative developmental erosion is not, by itself, equivalent to exhausted development.

In particular,

$$0 < g_t < g_t^w$$

implies positive absolute development together with relative erosion.

By contrast, the exhausted-development regime is defined separately by

$$g_t^A \leq 0,$$

meaning that positive calibrated development is no longer attainable.

Collapse-contingent reproduction choice. The relevant period-t choice compares two possible continuation branches. Under reproduction-preserving conduct, period-t actions generate a reproduced party-state position in period $t+1$, with continuation payoff

$$\mathscr{C}_{i,t}^{sys}(a_t^R).$$

Under reproduction-destroying conduct, the party-state fails to reproduce, the nomenklatura position is not reproduced, and collapse-contingent market exposure becomes relevant.

The two branches are institutionally asymmetric. Under reproduction, the nomenklatura remains a reproduced command body and serious all-cause disciplinary displacement affects only a small incidence of that body under Assumption 2.27. Under collapse, by contrast, the inherited nomenklatura position ceases to be reproduced for the entire body before any individual market-conversion outcome is realized.

I call this comparison a *collapse-contingent reproduction choice.* It is a coordination problem because the systemic consequence of one agent's conduct depends on the conduct of other nomenklatura agents. If a decisively weighted body withdraws developmental effort, withdraws from discipline-calibrating voice, or otherwise implements a reproduction-destroying coordinated deviation, party-state reproduction may fail.

The current realized systemic payoff

$$U_{i,t}^{sys}$$

remains part of the inherited state at the time the choice is made. It is not the payoff generated by the period-t reproduction decision.

3.1 Collapse-Contingent Market Exposure and the Non-Positive Collapse Alternative

Collapse first eliminates reproduction of the nomenklatura position as a party-state position. Post-collapse market conversion is analytically subsequent to that institutional loss.

Thus successful market conversion does not mean preservation of nomenklatura status. It means that a former nomenklatura agent obtains a different position in the post-collapse market environment.

The market alternative is collapse-contingent. For the nomenklatura, market conversion becomes relevant when the party-state fails to reproduce and protected political-administrative positions are destroyed. The resulting market condition is not treated as an institutionally neutral or risk-free outside option. The agent loses the protected systemic position and becomes exposed to a post-collapse market environment in which successful conversion is scarce, competition is uncontrolled, and transferable political-administrative advantages need not be sufficient to reproduce the former party-state position.

The relevant comparison is therefore not between a risky party-state position and a frictionless market benchmark. It is between a reproduced systemic position containing disciplinary exposure and a collapse-contingent market position containing conversion and competitive downside.

Let C_t^M denote the mass of agents competing for successful market positions under chaotic peripheral market conditions, and let B_t^M denote the number of successful conversion positions available. The scarcity of successful conversion is represented by

$$\lim_{t\to\infty} \frac{B_t^M}{C_t^M} = 0.$$

Let $a_i \geq 0$ denote agent i's transferable market advantage. For the non-outlier weighted body of the nomenklatura, assume this advantage is bounded:

$$0 \leq a_i \leq \bar{a} < \infty.$$

For the same non-outlier weighted body, the payoff from successful post-collapse conversion is also bounded:

$$0 \leq M_i^C \leq \bar{M}^C < \infty.$$

This boundedness condition allows for exceptional outliers. It states that the non-outlier body of the nomenklatura possesses a finite transferable market payoff, which prevents a large market payoff from offsetting the vanishing probability of successful post-collapse conversion.

The probability of successful post-collapse conversion is bounded by

$$\pi_i^C \leq a_i \frac{B_t^M}{C_t^M}.$$

While individual agents may succeed, successful post-collapse conversion is non-generalizable for the nomenklatura as a weighted body.

The collapse-contingent market exposure of agent i is

$$O_i^C = \pi_i^C M_i^C - (1 - \pi_i^C) L_i^C.$$

Let

$$\bar{O}_t^{C,w} = \frac{\sum_{i\in\mathcal{N}_t} w(h_i) O_i^C}{\sum_{i\in\mathcal{N}_t} w(h_i)}.$$

Let

$$A_t = \{i \in \mathcal{N}_t \,:\, O_i^C > 0\}$$

denote the set of outlier agents whose collapse-contingent market exposure is positive before disciplinary blocking.

Definition 3.3 (Non-Generalizable Outlier Set)**.** The outlier set A_t is non-generalizable when its hierarchical-weighted share in the nomenklatura tends to zero:

$$\lim_{t\to\infty} \mu_t^w(A_t) = \lim_{t\to\infty} \frac{\sum_{i\in A_t} w(h_i)}{\sum_{i\in\mathcal{N}_t} w(h_i)} = 0.$$

Agents in A_t are precisely the agents with positive collapse-contingent market exposure. The non-generalizability condition states that these agents cannot define the class-wide position of the nomenklatura.

Let

$$\mathcal{N}_t^0 = \mathcal{N}_t \setminus A_t$$

denote the non-outlier body of the nomenklatura.

Lemma 3.4 (Non-Generalizability of Successful Post-Collapse Market Conversion)**.** *Under chaotic post-collapse market conversion, the weighted average probability of successful post-collapse conversion tends to zero:*

$$\lim_{t\to\infty} \bar{\pi}_t^{C,w} = \lim_{t\to\infty} \frac{\sum_{i\in\mathcal{N}_t} w(h_i)\pi_i^C}{\sum_{i\in\mathcal{N}_t} w(h_i)} = 0.$$

Proof. Let

$$W_t = \sum_{i\in\mathcal{N}_t} w(h_i)$$

denote total hierarchical weight. Decompose the weighted average probability of successful post-collapse conversion into the non-outlier and outlier components:

$$\bar{\pi}_t^{C,w} = \frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)\pi_i^C}{W_t} + \frac{\sum_{i\in A_t} w(h_i)\pi_i^C}{W_t}.$$

For the non-outlier body $\mathcal{N}_t^0$,

$$\pi_i^C \le a_i \frac{B_t^M}{C_t^M}$$

and

$$a_i \le \bar{a}.$$

Therefore,

$$\pi_i^C \le \bar{a} \frac{B_t^M}{C_t^M}$$

for all $i \in \mathcal{N}_t^0$. Hence

$$0 \leq \frac{\sum_{i \in \mathcal{N}_t^0} w(h_i)\pi_i^C}{W_t} \leq \bar{a}\frac{B_t^M}{C_t^M},$$

and since

$$\lim_{t\to\infty} \frac{B_t^M}{C_t^M} = 0,$$

the squeeze theorem gives

$$\lim_{t\to\infty} \frac{\sum_{i \in \mathcal{N}_t^0} w(h_i)\pi_i^C}{W_t} = 0.$$

Thus, the non-outlier component tends to zero as $t \to \infty$.

For the outlier component, since $\pi_i^C \in [0,1]$,

$$0 \leq \frac{\sum_{i \in A_t} w(h_i)\pi_i^C}{W_t} \leq \mu_t^w(A_t).$$

By non-generalizability of the outlier set,

$$\lim_{t\to\infty} \mu_t^w(A_t) = 0.$$

Hence the outlier component also tends to zero.

Therefore,

$$\lim_{t\to\infty} \bar{\pi}_t^{C,w} = 0.$$

□

The result operates after the class-position loss established by Proposition 2.29. Collapse eliminates reproduction of the nomenklatura position for the inherited command body, while Lemma 3.4 establishes that successful conversion into alternative market positions is itself non-generalizable in hierarchical-weighted terms:

$$\bar{\pi}_t^{C,w} \to 0.$$

The two results therefore describe two distinct stages:

$$\boxed{\text{universal loss of the reproduced nomenklatura position} \longrightarrow \text{non-generalizable individual market conversion.}}$$

Assumption 3.5 (Positive Downside from Unsuccessful Post-Collapse Market Conversion)**.** Unsuccessful or adverse post-collapse market conversion imposes a nonnegative downside loss for all agents:

$$L_i^C \geq 0.$$

For the relevant non-outlier body of nomenklatura agents, unsuccessful or adverse post-collapse market exposure imposes a strictly positive downside loss. There exists

$$\underline{L} > 0$$

such that

$$L_i^C \geq \underline{L}$$

for all $i \in \mathcal{N}_t^0$.

Assumption 3.6 (Vanishing Outlier Contribution)**.** The hierarchical-weighted contribution of outlier successful-conversion payoffs vanishes asymptotically:

$$\lim_{t\to\infty} \frac{\sum_{i\in A_t} w(h_i)\pi_i^C M_i^C}{\sum_{i\in\mathcal{N}_t} w(h_i)} = 0.$$

Proposition 3.7 (Strictly Negative Class-Wide Collapse-Contingent Market Exposure)**.** *Under chaotic post-collapse market conversion and positive loss from failed conversion, collapse-contingent market exposure is strictly negative class-wide for the nomenklatura as a weighted body:*

$$\limsup_{t\to\infty} \bar{O}_t^{C,w} \leq -\underline{L} < 0.$$

Proof. For each agent,

$$O_i^C = \pi_i^C M_i^C - (1 - \pi_i^C) L_i^C.$$

Let

$$W_t = \sum_{i\in\mathcal{N}_t} w(h_i).$$

Then the weighted collapse-contingent market exposure is

$$\bar{O}_t^{C,w} = \frac{\sum_{i\in\mathcal{N}_t} w(h_i)\pi_i^C M_i^C}{W_t} - \frac{\sum_{i\in\mathcal{N}_t} w(h_i)(1-\pi_i^C)L_i^C}{W_t}.$$

Decompose the successful-conversion payoff into the non-outlier and outlier components:

$$\frac{\sum_{i\in\mathcal{N}_t} w(h_i)\pi_i^C M_i^C}{W_t} = \frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)\pi_i^C M_i^C}{W_t} + \frac{\sum_{i\in A_t} w(h_i)\pi_i^C M_i^C}{W_t}.$$

For the non-outlier body $\mathcal{N}_t^0$,

$$0 \leq M_i^C \leq \bar{M}^C < \infty.$$

Hence

$$\frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)\pi_i^C M_i^C}{W_t} \leq \bar{M}^C \frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)\pi_i^C}{W_t}.$$

By the non-outlier probability bound,

$$0 \leq \frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)\pi_i^C}{W_t} \leq \bar{a}\frac{B_t^M}{C_t^M},$$

so

$$\lim_{t\to\infty} \frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)\pi_i^C}{W_t} = 0.$$

Therefore, the non-outlier successful-conversion payoff vanishes:

$$\lim_{t\to\infty} \frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)\pi_i^C M_i^C}{W_t} = 0.$$

For the outlier component, Assumption 3.6 gives

$$\lim_{t\to\infty} \frac{\sum_{i\in A_t} w(h_i)\pi_i^C M_i^C}{W_t} = 0.$$

Hence the total weighted successful-conversion payoff vanishes.

For the non-outlier body

$$\mathcal{N}_t^0 = \mathcal{N}_t \setminus A_t,$$

use:

$$L_i^C \geq \underline{L} > 0,$$

and:

$$\pi_i^C \leq \bar{a}\frac{B_t^M}{C_t^M}.$$

Therefore:

$$1 - \pi_i^C \geq 1 - \bar{a}\frac{B_t^M}{C_t^M}.$$

Hence:

$$\frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)(1-\pi_i^C)L_i^C}{W_t} \geq \underline{L}\left(1 - \bar{a}\frac{B_t^M}{C_t^M}\right)\frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)}{W_t}.$$

Using:

$$\frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)}{W_t} = 1 - \mu_t^w(A_t),$$

obtain:

$$\frac{\sum_{i\in\mathcal{N}_t^0} w(h_i)(1-\pi_i^C)L_i^C}{W_t} \geq \underline{L}\left(1 - \bar{a}\frac{B_t^M}{C_t^M}\right)(1-\mu_t^w(A_t)).$$

Since:

$$\lim_{t\to\infty} \frac{B_t^M}{C_t^M} = 0$$

and:

$$\lim_{t\to\infty} \mu_t^w(A_t) = 0,$$

conclude:

$$\liminf_{t\to\infty} \frac{\sum_{i\in\mathcal{N}_t} w(h_i)(1-\pi_i^C)L_i^C}{W_t} \geq \underline{L}.$$

The outlier failed-conversion loss contribution may be omitted from the lower bound because it is nonnegative.

Combining this with the vanishing weighted successful-conversion payoff gives:

$$\limsup_{t\to\infty} \bar{O}_t^{C,w} \leq -\underline{L} < 0.$$

Outliers with $O_i^C > 0$ may exist, but when their weighted share and successful-conversion contribution are non-generalizable, they fail to define the class-wide position of the nomenklatura. □

Corollary 3.8 (Eventual Strict Negativity of Class-Wide Collapse-Contingent Market Exposure)**.** *Under the conditions of Proposition 3.7, there exists a finite historical threshold*

$$T^C < \infty$$

such that, for every

$$t \geq T^C,$$

the hierarchical-weighted class-wide collapse-contingent market exposure satisfies

$$\boxed{\bar{O}_t^{C,w} \leq -\frac{\underline{L}}{2} < 0.}$$

Proof. By Proposition 3.7,

$$\limsup_{t\to\infty} \bar{O}_t^{C,w} \leq -\underline{L} < 0.$$

Choose

$$\varepsilon = \frac{\underline{L}}{2} > 0.$$

By the definition of the limit superior, there exists

$$T^C < \infty$$

such that, for every

$$t \geq T^C,$$

$$\bar{O}_t^{C,w} < \limsup_{s\to\infty} \bar{O}_s^{C,w} + \frac{\underline{L}}{2}.$$

Since

$$\limsup_{s\to\infty} \bar{O}_s^{C,w} \leq -\underline{L},$$

it follows that

$$\bar{O}_t^{C,w} < -\frac{\underline{L}}{2} < 0.$$

Hence, in particular,

$$\bar{O}_t^{C,w} \leq -\frac{\underline{L}}{2} < 0$$

for every

$$t \geq T^C.$$

□

Proposition 3.7 establishes the asymptotic class-wide result, while Corollary 3.8 converts that result into a finite historical statement. After the finite threshold

$$T^C,$$

the hierarchical-weighted collapse-contingent market exposure of the nomenklatura is strictly negative class-wide. This provides a material basis for rational systemic loyalty independent of ideological commitment or moral attachment to the party-state. Individual positive-conversion outliers may continue to exist, but they do not define the weighted class-wide position.

3.2 Developmental Alignment of Nomenklatura Interest

The previous proposition establishes that post-collapse market conversion is not a generalizable alternative for the nomenklatura as a weighted body. The next step is to show why, under positive-development conditions, the nomenklatura is tied to the development of its own party-state.

The alignment of nomenklatura interest is structural: the systemic class benefit of the nomenklatura increases with the developedness of the party-state system:

$$\frac{\partial Z(D_t, h_i)}{\partial D_t} > 0.$$

Systemic developedness evolves through the historical efficiency norm:

$$D_{t+1} = D_t(1 + g_t).$$

Developmental effort matters because aggregate developmental effort moves the historical efficiency norm toward the frontier efficiency norm. The function

$$\Gamma(X_t, g_t^{\max})$$

specifies the frontier-directed component of the historical efficiency norm generated by aggregate developmental effort.

The developmental-alignment mechanism also has a relative dimension. Since the advanced world developedness frontier itself continues to move according to

$$g_t^w > 0,$$

higher party-state growth not only raises absolute systemic developedness but also determines whether the party-state closes or widens its developmental gap relative to that frontier.

Whenever realized calibrated growth satisfies

$$g_t > g_t^w,$$

developmental alignment produces catching-up development and

$$\rho_{t+1} > \rho_t.$$

When
$$0 < g_t < g_t^w,$$
developmental alignment still produces positive absolute growth, but the party-state's relative position declines.

A period in which
$$g_t < g_t^w$$
produces relative developmental erosion, while a historically accumulated stock of developedness and command capacity may allow the party-state to continue reproducing as its relative position gradually declines.

Definition 3.9 (Frontier-Directed Developmental Function)**.** The *frontier-directed developmental function* is
$$\Gamma(X_t, g_t^{\max}),$$
where
$$X_t = \sum_{i \in \mathcal{N}_t} w(h_i) x_{i,t}.$$
It is increasing in aggregate developmental effort:
$$\frac{\partial \Gamma(X_t, g_t^{\max})}{\partial X_t} > 0,$$
and bounded above by the frontier efficiency norm:
$$0 \leq \Gamma(X_t, g_t^{\max}) \leq g_t^{\max}.$$

Definition 3.10 (Role-Bound Developmental Effort)**.** For each nomenklatura agent i, developmental effort is bounded by the maximum level accessible within his institutional role:
$$0 \leq x_{i,t} \leq \bar{x}_i(g_t^{\max}, h_i).$$
The upper bound $\bar{x}_i(g_t^{\max}, h_i)$ is the maximum role-bound developmental effort available to agent i under the frontier efficiency norm and his hierarchical position.

The role-bound maximum is a finite quantity defined by the institutional scope of the position. An agent's office, authority, administrative access, working obligations, and command capacity define the maximum effort he can apply within the party-state.

The maximum accessible aggregate developmental effort is
$$\bar{X}_t = \sum_{i \in \mathcal{N}_t} w(h_i) \bar{x}_i(g_t^{\max}, h_i).$$

Assume that, for fixed $g_t^{\max}$, the function
$$X \mapsto \Gamma(X, g_t^{\max})$$

is continuous on the accessible effort interval

$$[0, \bar{X}_t].$$

At maximum accessible aggregate developmental effort, the frontier-directed component reaches the frontier efficiency norm:

$$\Gamma(\bar{X}_t, g_t^{\max}) = g_t^{\max}.$$

Role-bounded developmental effort and comparative effort cost. The developmental-effort variable

$$x_{i,t}$$

does not represent unrestricted labor supply or the number of hours worked by agent *i*. It represents the developmental contribution that can be exercised through the agent's institutional role. The accessible choice set is therefore bounded by

$$0 \leq x_{i,t} \leq \bar{x}_i(g_t^{\max}, h_i),$$

where

$$\bar{x}_i(g_t^{\max}, h_i)$$

is determined by the scope of the office, its authority, administrative access, assigned functions, and the historically available frontier efficiency norm.

Accordingly, the model does not determine developmental effort through a standard labor-supply trade-off of the form

$$\text{systemic return} - c_i(x_{i,t}).$$

The upper bound

$$\bar{x}_i(g_t^{\max}, h_i)$$

is an institutional and functional bound, not an interior optimum generated by increasing marginal disutility of labor. Proposition 3.11 establishes that, over this role-bounded accessible interval, the developmental component of the agent's next-period systemic position is strictly increasing in

$$x_{i,t}.$$

Hence the developmental optimum lies at the role-bound upper boundary.

This modeling choice does not require the claim that effort is literally costless. It means only that ordinary effort burden is not the mechanism that determines the developmental optimum studied here. The relevant object is the maximum developmental contribution available through a given command role.

Comparative effort cost remains relevant to a separate objection concerning collapse-contingent market conversion. Exit through systemic collapse is not treated as a lower-effort alternative that automatically dominates role-bound developmental conduct. Let

$$c_i^{sys}\big(\bar{x}_i(g_t^{\max}, h_i)\big)$$

denote the effort burden associated with fulfilling the maximum accessible developmental function inside the party-state, and let

$$c_i^C$$

denote the effort burden associated with attempting post-collapse market conversion. The maintained comparative condition is

$$c_i^C \geq c_i^{sys}\big(\bar{x}_i(g_t^{\max}, h_i)\big).$$

Thus collapse-contingent market conversion cannot be justified within the model merely as an escape into a systematically lower-effort condition. This comparative condition is auxiliary to the developmental-alignment result; it does not enter the marginal determination of

$$x_{i,t}^*.$$

One-period forward-looking developmental choice. Under the timing convention of Subsection 2.2.5, the systemic state observed at time t is already realized when period-t developmental effort is chosen. The developmental consequence of

$$x_{i,t}$$

therefore enters the state realized at time

$$t + 1.$$

For the developmental-effort choice, the relevant systemic return is the next-period value of the agent's reproduced party-state position. Holding the realized period-t disciplinary environment

$$P_t$$

and the period-t actions of the other nomenklatura agents fixed, agent i's effort changes aggregate developmental effort and thereby changes next-period developedness, relative developedness, and stability.

Accordingly, the developmental component of the agent's one-period forward-looking objective is

$$S_{t+1} Z(D_{t+1}, h_i).$$

The deviation-rent and disciplinary-exposure components of systemic utility remain distinct components of the broader choice problem and are analyzed in their dedicated sections.

Proposition 3.11 (Developmental Alignment of Nomenklatura Interest)**.** *Under the positive-development regime, consider a system-loyal nomenklatura agent i choosing period-t developmental effort*

$$x_{i,t} \in [0, \bar{x}_i(g_t^{\max}, h_i)],$$

holding the realized disciplinary level

$$P_t$$

and the period-t developmental efforts of all other agents fixed.

The next-period stability-adjusted systemic class benefit

$$S_{t+1}Z(D_{t+1}, h_i)$$

is strictly increasing in

$$x_{i,t}.$$

Consequently, the developmental component of the agent's one-period forward-looking choice is maximized at the maximum accessible level of role-bound developmental effort:

$$\boxed{x_{i,t}^{*} = \bar{x}_i(g_t^{\max}, h_i).}$$

When this developmental alignment holds throughout the system-loyal reproduction-preserving body of the nomenklatura, aggregate developmental effort reaches its maximum accessible level:

$$X_t^{*} = \bar{X}_t.$$

At this level,

$$\Gamma(X_t^{*}, g_t^{\max}) = \Gamma(\bar{X}_t, g_t^{\max}) = g_t^{\max}.$$

Proof. Fix the realized period-t disciplinary level

$$P_t$$

and the developmental efforts of all nomenklatura agents other than agent i.

Aggregate developmental effort is

$$X_t = \sum_{j \in \mathcal{N}_t} w(h_j) x_{j,t},$$

so

$$\frac{\partial X_t}{\partial x_{i,t}} = w(h_i) > 0.$$

The frontier-directed developmental function is strictly increasing in aggregate developmental effort:

$$\frac{\partial \Gamma(X_t, g_t^{\max})}{\partial X_t} > 0.$$

Hence an increase in

$$x_{i,t}$$

strictly raises the frontier-directed developmental component generated during period t.

For a fixed realized disciplinary level

$$P_t,$$

the period-t deviation-harm and disciplinary-cost terms are held fixed with respect to this individual developmental-effort comparison. Therefore, higher aggregate developmental effort raises the calibrated historical efficiency norm:

$$\frac{\partial g_t(X_t, P_t)}{\partial x_{i,t}} = \frac{\partial \Gamma(X_t, g_t^{\max})}{\partial X_t} w(h_i) > 0.$$

Next-period systemic developedness is

$$D_{t+1} = D_t \left[1 + g_t(X_t, P_t)\right].$$

Since

$$D_t > 0,$$

it follows that

$$\frac{\partial D_{t+1}}{\partial x_{i,t}} = D_t \frac{\partial g_t(X_t, P_t)}{\partial x_{i,t}} > 0.$$

Relative developedness is

$$\rho_{t+1} = \frac{D_{t+1}}{D_{t+1}^w}.$$

Because

$$D_{t+1}^w > 0,$$

we obtain

$$\frac{\partial \rho_{t+1}}{\partial x_{i,t}} = \frac{1}{D_{t+1}^w} \frac{\partial D_{t+1}}{\partial x_{i,t}} > 0.$$

Under the realized period-t disciplinary environment,

$$S_{t+1} = H(\rho_{t+1}, P_t),$$

with

$$H_\rho = \frac{\partial H(\rho_{t+1}, P_t)}{\partial \rho_{t+1}} > 0.$$

Therefore,

$$\frac{\partial S_{t+1}}{\partial x_{i,t}} = H_\rho \frac{\partial \rho_{t+1}}{\partial x_{i,t}} > 0.$$

Now consider the next-period stability-adjusted systemic class benefit:

$$S_{t+1} Z(D_{t+1}, h_i).$$

Differentiating with respect to period-t developmental effort gives

$$\frac{\partial}{\partial x_{i,t}} \left[S_{t+1} Z(D_{t+1}, h_i)\right] = Z(D_{t+1}, h_i) \frac{\partial S_{t+1}}{\partial x_{i,t}} + S_{t+1} Z_D(D_{t+1}, h_i) \frac{\partial D_{t+1}}{\partial x_{i,t}}.$$

By construction,

$$Z(D_{t+1}, h_i) > 0, \qquad Z_D(D_{t+1}, h_i) > 0,$$

and the preceding steps establish

$$\frac{\partial S_{t+1}}{\partial x_{i,t}} > 0, \qquad \frac{\partial D_{t+1}}{\partial x_{i,t}} > 0.$$

Hence

$$\boxed{\frac{\partial}{\partial x_{i,t}} \left[S_{t+1} Z(D_{t+1}, h_i) \right] > 0.}$$

The next-period stability-adjusted systemic class benefit is therefore strictly increasing throughout the accessible role-bound effort interval

$$\left[0, \bar{x}_i(g_t^{\max}, h_i)\right].$$

Its maximum is attained at the upper boundary:

$$x_{i,t}^* = \bar{x}_i(g_t^{\max}, h_i).$$

If developmental alignment holds throughout the system-loyal reproduction-preserving body, aggregation gives

$$X_t^* = \sum_{i \in \mathcal{N}_t} w(h_i) x_{i,t}^* = \sum_{i \in \mathcal{N}_t} w(h_i) \bar{x}_i(g_t^{\max}, h_i) = \bar{X}_t.$$

By the definition of the maximum accessible aggregate developmental effort,

$$\Gamma(\bar{X}_t, g_t^{\max}) = g_t^{\max}.$$

Therefore,

$$\Gamma(X_t^*, g_t^{\max}) = \Gamma(\bar{X}_t, g_t^{\max}) = g_t^{\max}.$$

Thus period-t maximum role-bound developmental effort is aligned with the agent's next-period reproduced systemic position: greater current developmental effort raises next-period developedness and stability, and therefore raises the next-period stability-adjusted systemic class benefit attached to the agent's nomenklatura position. □

Proposition 3.11 establishes the full-effort developmental benchmark. The next subsection evaluates departures from that benchmark, distinguishing localized shortfalls from generalized strategically significant effort withdrawal.

3.3 Localized and Generalized Free Riding

Developmental Alignment establishes maximum accessible role-bound effort as the reproduction-preserving developmental benchmark. This benchmark does not require every realized micro-level action to coincide mechanically with its optimum in every period. Individual agents or small subsets of agents may display localized underperformance, residual shirking, implementation error, or other off-benchmark effort shortfalls while the aggregate developmental configuration remains arbitrarily close to the full-effort benchmark.

Localized free riding is therefore treated here as a possible small realized deviation from

$$X_t^* = \bar{X}_t,$$

not as a second rational developmental optimum. The relevant distinction is between a localized aggregate shortfall whose macro-systemic effect is correspondingly small and generalized strategically significant effort withdrawal by a sufficiently important body of nomenklatura agents.

No compensating effort above the role-bound maxima is required. Agents outside the localized free-riding set may remain at their own role-bound maxima,

$$x_{j,t} = \bar{x}_j(g_t^{\max}, h_j),$$

while aggregate effort is slightly below

$$\bar{X}_t$$

because of the localized shortfall itself.

To see this, consider a subset of nomenklatura agents

$$B_t \subseteq \mathcal{N}_t$$

that attempts to free ride by choosing

$$x_{i,t} < \bar{x}_i(g_t^{\max}, h_i) \qquad \text{for } i \in B_t,$$

while the remaining agents choose their role-bound maximum. Aggregate developmental effort then becomes

$$X_t(B_t) = \bar{X}_t - \Delta X_t(B_t),$$

where

$$\Delta X_t(B_t) = \sum_{i \in B_t} w(h_i) \left[\bar{x}_i(g_t^{\max}, h_i) - x_{i,t}\right] > 0$$

whenever the free-riding set has positive hierarchical weight and reduces effort.

The quantity

$$\Delta X_t(B_t)$$

is the aggregate developmental-effort shortfall relative to the full role-bound benchmark. A positive shortfall always lowers aggregate effort strictly, but its systemic importance depends on its magnitude. In particular, a very small

$$\Delta X_t(B_t)$$

need not produce a regime change or any economically significant change in the macro-systemic trajectory.

Since the frontier-directed developmental function is increasing in aggregate developmental effort,

$$\frac{\partial \Gamma(X_t, g_t^{\max})}{\partial X_t} > 0,$$

it follows that

$$\Gamma(X_t(B_t), g_t^{\max}) < \Gamma(\bar{X}_t, g_t^{\max}).$$

For a given disciplinary level, the calibrated historical efficiency norm therefore falls:

$$g_t(X_t(B_t), P_t) < g_t(\bar{X}_t, P_t).$$

Hence every positive effort shortfall lowers the developmental path relative to the exact full-effort benchmark. This strict comparison does not imply that every localized shortfall is macroscopically important. When the shortfall is sufficiently small, the resulting developmental loss may itself be arbitrarily small.

Lemma 3.12 (Macroscopic Smallness of Localized Developmental Shortfalls)**.** *Fix period t and realized discipline*

$$P_t.$$

Because

$$X \mapsto \Gamma(X, g_t^{\max})$$

is continuous at

$$\bar{X}_t,$$

for every

$$\eta > 0$$

there exists

$$\varepsilon_t^X(\eta) > 0$$

such that

$$0 < \Delta X_t(B_t) < \varepsilon_t^X(\eta)$$

implies

$$0 < g_t(\bar{X}_t, P_t) - g_t(X_t(B_t), P_t) < \eta.$$

Consequently, sufficiently small localized effort shortfalls generate correspondingly small deviations from the full-effort paths of next-period systemic developedness, relative developedness, and systemic stability.

Proof. At fixed

$$P_t,$$

the calibrated historical efficiency norm is

$$g_t(X, P_t) = \Gamma(X, g_t^{\max}) - E_t(P_t) - K(P_t).$$

Hence

$$g_t(\bar{X}_t, P_t) - g_t(X_t(B_t), P_t) = \Gamma(\bar{X}_t, g_t^{\max}) - \Gamma(X_t(B_t), g_t^{\max}).$$

Since

$$X_t(B_t) = \bar{X}_t - \Delta X_t(B_t)$$

and

$$X \mapsto \Gamma(X, g_t^{\max})$$

is continuous at

$$\bar{X}_t,$$

for every

$$\eta > 0$$

there exists

$$\varepsilon_t^X(\eta) > 0$$

such that

$$0 < \Delta X_t(B_t) < \varepsilon_t^X(\eta)$$

implies

$$\left|\Gamma(\bar{X}_t, g_t^{\max}) - \Gamma(X_t(B_t), g_t^{\max})\right| < \eta.$$

Strict monotonicity of

$$\Gamma$$

gives the strict lower inequality, so

$$0 < g_t(\bar{X}_t, P_t) - g_t(X_t(B_t), P_t) < \eta.$$

Next-period systemic developedness satisfies

$$D_{t+1}(X, P_t) = D_t \left[1 + g_t(X, P_t)\right].$$

Therefore,

$$0 < D_{t+1}^* - D_{t+1}(B_t) < D_t\eta.$$

Since

$$\rho_{t+1} = \frac{D_{t+1}}{D_{t+1}^w},$$

it follows that

$$0 < \rho_{t+1}^* - \rho_{t+1}(B_t) < \frac{D_t}{D_{t+1}^w}\eta.$$

Finally,

$$S_{t+1} = H(\rho_{t+1}, P_t),$$

and H is continuous in relative developedness on the modeled domain. Hence

$$\rho_{t+1}(B_t) \to \rho_{t+1}^*$$

as

$$\Delta X_t(B_t) \to 0$$

implies

$$S_{t+1}(B_t) \to S_{t+1}^*.$$

Thus localized developmental shortfalls can generate arbitrarily small macro-systemic deviations from the full-effort benchmark. □

The lemma distinguishes a strict mathematical loss from a systemically significant loss. A localized shortfall may reduce calibrated growth from its exact full-effort value while leaving the realized growth regime, reproduction condition, and broader macro-systemic trajectory effectively unchanged. In this sense, localized underperformance can appear as a small fluctuation around the full-effort developmental benchmark.

The result does not rely on compensating super-maximal effort by other nomenklatura agents. No agent is required to exceed

$$\bar{x}_i(g_t^{\max}, h_i).$$

The macro-systemic effect is small because the aggregate shortfall itself is small.

The effect is not external to the free-riding agents. Lower aggregate developmental effort lowers next-period systemic developedness relative to the full-effort path:

$$D_{t+1}(B_t) < D_{t+1}^*.$$

It also lowers relative developedness,

$$\rho_{t+1}(B_t) < \rho_{t+1}^*,$$

and, since systemic stability is increasing in relative developedness,

$$H_\rho > 0,$$

it weakens systemic stability:

$$S_{t+1}(B_t) < S_{t+1}^*.$$

Because systemic class benefit is increasing in developedness,

$$Z_D > 0,$$

the free-riding agents themselves face a reduction in the systemic class benefit attached to their own positions:

$$Z(D_{t+1}(B_t), h_i) < Z(D_{t+1}^*, h_i).$$

These inequalities identify the direction of the effect. Lemma 3.12 identifies its magnitude near the full-effort benchmark: for sufficiently small localized shortfalls, the resulting losses are correspondingly small. The strategic objection becomes substantively important when effort withdrawal is generalized or otherwise large enough to create a non-negligible change in the developmental or reproductive path.

The exclusion of generalized strategically significant free riding requires agents to understand the systemic consequences of the collective action profile they are considering. The model imposes this in the standard common-knowledge sense. Nomenklatura agents are rational, rationality is common knowledge, and the model-relevant systemic transition structure is common knowledge. Hence a system-loyal agent evaluates a contemplated generalized developmental-effort profile in light of its known consequences for next-period development and party-state reproduction.

Assumption 3.13 (Common Knowledge of Rationality and Systemic Transition Structure)**.** Nomenklatura agents are rational, and their rationality is common knowledge. The model-relevant structure of the party-state is also common knowledge among the agents participating in the reproduction decision. In particular, the following are common knowledge:

(i) the role-bound feasible developmental-effort sets,

$$0 \le x_{i,t} \le \bar{x}_i(g_t^{\max}, h_i);$$

(ii) the realized period-t systemic conditions relevant to the transition, including

$$D_t, \qquad \rho_t, \qquad P_t, \qquad S_t;$$

(iii) the systemic transition structure

$$(x_{i,t})_{i \in \mathcal{N}_t} \longrightarrow X_t \longrightarrow \Gamma(X_t, g_t^{\max}) \longrightarrow g_t(X_t, P_t) \longrightarrow D_{t+1} \longrightarrow \rho_{t+1} \longrightarrow S_{t+1};$$

(iv) the critical reproduction threshold

$$S_c.$$

Accordingly, for any contemplated aggregate developmental-effort profile

$$X_t,$$

agents can evaluate the induced next-period systemic developedness

$$D_{t+1}(X_t, P_t),$$

relative developedness

$$\rho_{t+1}(X_t, P_t),$$

and systemic stability

$$S_{t+1}(X_t, P_t),$$

and can determine whether the contemplated profile preserves the reproduction condition

$$S_{t+1}(X_t, P_t) \ge S_c$$

or violates it,

$$S_{t+1}(X_t, P_t) < S_c.$$

Common knowledge is used here in the standard model-theoretic sense: each relevant agent knows the strategic and systemic structure, knows that the other relevant agents know it, and so on. The assumption does not require historical agents to represent that structure in the analytical notation used by the model.

To formalize the evaluation of free riding, consider a contemplated effort-withdrawal profile by a subset of nomenklatura agents $B_t \subseteq \mathcal{N}_t$. An individual effort reduction $x_{i,t} < \bar{x}_i(g_t^{\max}, h_i)$ may be locally small. However, agents evaluate the counterfactual generalization of analogous effort withdrawal across a decisively weighted body $B_t \subseteq \mathcal{N}_t$. Let such a generalized free-riding profile yield an aggregate developmental effort

$$\widehat{X}_t < \bar{X}_t.$$

By Assumption 3.13, agents evaluate the systemic consequences of $\widehat{X}_t$ across two mutually exclusive and exhaustive cases:

- **Case I (Reproduction-Feasible Generalization):**

  $$S_{t+1}(\widehat{X}_t, P_t) \geq S_c.$$

  The party-state reproduces, but the reduced aggregate effort $\widehat{X}_t$ depresses the frontier-directed historical efficiency component, $\Gamma(\widehat{X}_t, g_t^{\max}) < \Gamma(\bar{X}_t, g_t^{\max})$, yielding a lower calibrated development path $g_t(\widehat{X}_t, P_t) < g_t(\bar{X}_t, P_t)$. This induces lower next-period systemic developedness $D_{t+1}(\widehat{X}_t) < D_{t+1}^*$, lower relative developedness $\rho_{t+1}(\widehat{X}_t) < \rho_{t+1}^*$, weaker stability $S_{t+1}(\widehat{X}_t) < S_{t+1}^*$, and lower systemic class benefit:

  $$Z(D_{t+1}(\widehat{X}_t), h_i) < Z(D_{t+1}^*, h_i).$$

  By Proposition 3.11 (Developmental Alignment), system-loyal agents face a positive structural incentive to choose full role-bound effort $x_{i,t}^* = \bar{x}_i(g_t^{\max}, h_i)$ because their reproduced class benefit depends monotonically on systemic developedness and stability. Generalized free riding thus yields a strictly inferior reproduction outcome compared to full role-bound developmental effort.

- **Case II (Reproduction-Destroying Generalization):**

  $$S_{t+1}(\widehat{X}_t, P_t) < S_c.$$

  The resulting counterfactual outcome is the collapse branch, which triggers collapse-contingent market conversion.

  Let

  $$a_t^R$$

  denote the reproduction-preserving benchmark against which the generalized withdrawal is evaluated. For the system-loyal reproduction-preserving body, impose the one-period continuation-loyalty condition

  $$\mathscr{C}_{i,t}^{sys}(a_t^R) \geq O_i^C.$$

  Thus the relevant ex ante comparison is between the systemic payoff attached to reproduction of the party-state in period $t+1$ and the collapse-contingent payoff generated if the contemplated generalized withdrawal destroys reproduction. Because the reproduction consequence is evaluated ex ante under Assumption 3.13, the contemplated generalized profile is recognized before implementation as reproduction-destroying.

This ex ante case analysis yields the central exclusion result for generalized free riding.

Proposition 3.14 (Ex Ante Exclusion of Generalized Free Riding). *Under the positive-development regime, Developmental Alignment (Proposition 3.11), Common Knowledge of Rationality and Systemic Transition Structure (Assumption 3.13), and one-period continuation loyalty*

$$\mathscr{C}_{i,t}^{sys}(a_t^R) \geq O_i^C$$

for the reproduction-preserving body, a generalized free-riding profile by a decisively weighted body $B_t \subseteq \mathcal{N}_t$ cannot constitute a rational reproduction-preserving class strategy for the system-loyal nomenklatura.

Proof. Consider any contemplated effort-withdrawal profile by a decisively weighted body $B_t \subseteq \mathcal{N}_t$ resulting in generalized aggregate effort $\widehat{X}_t < \bar{X}_t$. By Assumption 3.13, rationality and the systemic transition structure are common knowledge, so system-loyal agents evaluate the counterfactual systemic outcome of

$$\widehat{X}_t$$

ex ante.

If $S_{t+1}(\widehat{X}_t, P_t) \geq S_c$ (Case I), party-state reproduction is preserved, but the reduction in aggregate effort lowers frontier-directed growth $\Gamma(\widehat{X}_t, g_t^{\max})$, calibrated historical growth $g_t(\widehat{X}_t, P_t)$, next-period developedness $D_{t+1}(\widehat{X}_t) < D_{t+1}^*$, and systemic stability $S_{t+1}(\widehat{X}_t) < S_{t+1}^*$. Because $Z_D > 0$, each agent's systemic class benefit satisfies $Z(D_{t+1}(\widehat{X}_t), h_i) < Z(D_{t+1}^*, h_i)$. By Proposition 3.11, the developmental component of the system-loyal agent's next-period reproduced position is maximized at role-bound effort

$$x_{i,t}^* = \bar{x}_i(g_t^{\max}, h_i).$$

Hence, on the reproduction-feasible domain, $\widehat{X}_t$ yields a strictly inferior developmental outcome relative to $\bar{X}_t$.

If

$$S_{t+1}(\widehat{X}_t, P_t) < S_c$$

(Case II), party-state reproduction fails and the collapse-contingent payoff

$$O_i^C$$

becomes relevant.

Let

$$a_t^R$$

denote the reproduction-preserving benchmark. By the maintained one-period continuation-loyalty condition,

$$\mathscr{C}_{i,t}^{sys}(a_t^R) \geq O_i^C.$$

Hence the reproduction-destroying generalized profile does not dominate the reproduced next-period systemic position for the continuation-loyal body. Because its reproduction consequence is evaluated ex ante under Assumption 3.13, it is excluded prior to implementation as a rational reproduction-preserving class choice.

Thus, in neither case can generalized free riding by a decisively weighted body constitute a rational reproduction-preserving class strategy for the system-loyal nomenklatura under the maintained developmental-alignment, common-knowledge, and loyalty conditions. □

The ex ante logic of this result follows from common knowledge rather than from an assumption that agents literally possess the model's equations in analytical form. Rational agents know the systemic transition structure and know that the other relevant agents know it. Consequently, when generalized strategically significant effort withdrawal is contemplated, its developmental and reproductive consequences enter the decision problem before the profile is implemented. The model therefore requires common knowledge of the relevant strategic structure, not literal omniscience.

Proposition 3.14 excludes generalized strategically significant free riding by a decisively weighted reproduction-preserving body.

Localized underperformance therefore remains possible as an off-benchmark realization. It may take the form of local free riding, marginal underperformance, residual shirking, implementation error, or isolated deviation. Such behavior does not constitute an alternative class-wide developmental optimum established by the model.

When its aggregate shortfall is small, its macro-systemic effect is correspondingly small by Lemma 3.12. When similar underperformance becomes sufficiently widespread, hierarchically weighted, or persistent to produce a strategically significant aggregate shortfall, it ceases to be merely localized and enters the generalized effort-withdrawal problem analyzed above.

Thus the model distinguishes:

$$\boxed{\text{localized off-benchmark shortfall} \neq \text{generalized strategically significant effort withdrawal.}}$$

3.4 Deviation, Discipline Costs, and the Calibrated Historical Efficiency Norm

Developmental effort moves the historical efficiency norm toward the frontier efficiency norm. However, the historical efficiency norm is calibrated by two losses internal to the model: harm from deviation and the cost of discipline.

Definition 3.15 (Harm from Deviation)**.** *Harm from deviation* is the aggregate reduction in systemic growth generated by residual incentives and realized attempts to deviate from systemic reproduction.

Assumption 3.16 (Deviation-Harm Monotonicity)**.** Aggregate harm from deviation is generated by the vector of residual deviation rents:

$$E_t(P) = \mathscr{E}_t\left((r_i(P, D_t, h_i))_{i \in \mathscr{N}_t}\right),$$

where

$$\mathscr{E}_t : \mathbb{R}_+^{n_t} \to \mathbb{R}_+$$

is weakly increasing in every argument. Thus, if

$$r_i' \le r_i \qquad \text{for every } i \in \mathscr{N}_t,$$

then

$$\mathscr{E}_t((r_i')_{i \in \mathscr{N}_t}) \le \mathscr{E}_t((r_i)_{i \in \mathscr{N}_t}).$$

Formally,

$$E_t(P_t) \geq 0$$

denotes aggregate harm from deviation under disciplinary level P_t.

Definition 3.17 (Cost of Discipline)**.** *Cost of discipline* is the resource and systemic cost of maintaining the party-state's disciplinary regime.

Formally,

$$K(P_t) \geq 0$$

denotes the cost of discipline under disciplinary level P_t.

Assumption 3.18 (Costliness of Discipline)**.** The cost of discipline is increasing in the level of discipline:

$$\frac{\partial K(P_t)}{\partial P_t} > 0.$$

Definition 3.19 (Calibrated Historical Efficiency Norm)**.** The *calibrated historical efficiency norm* is the frontier-directed component generated by aggregate developmental effort, calibrated by harm from deviation and the cost of discipline:

$$g_t(X_t, P_t) = \Gamma(X_t, g_t^{\max}) - E_t(P_t) - K(P_t).$$

This equation incorporates all loss terms considered at this stage.

Any admissible effort-discipline configuration used to generate next-period systemic developedness is restricted to $g_t(X_t, P_t) > -1$, ensuring $D_{t+1} > 0$.

3.5 Discipline and Residual Deviation

The reduction in deviation under discipline is a derived result rather than a raw assumption. It follows from the definition of residual deviation rent and the effect of discipline on detection and sanction probability.

Recall that the expected return from deviation is

$$\mathbb{E}[R_i(e_i, P_t, D_t, h_i)] = (1 - q_i(P_t, h_i))b_i(e_i, D_t, h_i) - q_i(P_t, h_i)f_i(e_i, h_i).$$

Equivalently,

$$\mathbb{E}[R_i(e_i, P_t, D_t, h_i)] = b_i(e_i, D_t, h_i) - q_i(P_t, h_i)\big[b_i(e_i, D_t, h_i) + f_i(e_i, h_i)\big].$$

Since discipline raises the probability of detection or sanction,

$$\frac{\partial q_i(P_t, h_i)}{\partial P_t} > 0.$$

Lemma 3.20 (Discipline Reduces Residual Deviation Rent)**.** *Under the maintained conditions*

$$b_i(e_i, D_t, h_i) \geq 0, \qquad f_i(e_i, h_i) \geq 0,$$

for every admissible $e_i > 0$*, and*

$$\frac{\partial q_i(P_t, h_i)}{\partial P_t} > 0,$$

then for any $P' \geq P$*,*

$$r_i(P', D_t, h_i) \leq r_i(P, D_t, h_i).$$

Where r_i *is differentiable with respect to P, this implies*

$$\frac{\partial r_i(P, D_t, h_i)}{\partial P} \leq 0.$$

Proof. For every $e_i > 0$,

$$P' \geq P \quad \Rightarrow \quad q_i(P', h_i) \geq q_i(P, h_i),$$

and therefore, under the condition $b_i + f_i \geq 0$,

$$\mathbb{E}[R_i(e_i, P', D_t, h_i)] \leq \mathbb{E}[R_i(e_i, P, D_t, h_i)].$$

The residual deviation rent is

$$r_i(P_t, D_t, h_i) = \max\left\{0, \sup_{e_i>0} \mathbb{E}[R_i(e_i, P_t, D_t, h_i)]\right\}.$$

Taking the supremum over $e_i > 0$ and applying the outer $\max\{0, \cdot\}$ preserves the weak inequality:

$$r_i(P', D_t, h_i) \leq r_i(P, D_t, h_i).$$

□

Corollary 3.21 (Discipline Reduces Harm from Deviation)**.** *Under Assumption 3.16, for any*

$$P' \geq P,$$

aggregate harm from deviation satisfies

$$E_t(P') \leq E_t(P).$$

Where E_t *is differentiable in P, this implies*

$$\frac{\partial E_t(P)}{\partial P} \leq 0.$$

Proof. By Lemma 3.20,

$$P' \geq P \quad \Rightarrow \quad r_i(P', D_t, h_i) \leq r_i(P, D_t, h_i)$$

for every $i \in \mathcal{N}_t$.

By Assumption 3.16, $\mathcal{E}_t$ is weakly increasing in every residual-rent argument. Therefore,

$$E_t(P') = \mathcal{E}_t\left((r_i(P', D_t, h_i))_{i\in\mathcal{N}_t}\right) \leq \mathcal{E}_t\left((r_i(P, D_t, h_i))_{i\in\mathcal{N}_t}\right) = E_t(P).$$

Hence, where differentiable,

$$\frac{\partial E_t(P)}{\partial P} \leq 0.$$

□

The distinction between catching-up development and positive calibrated development is also important for disciplinary capacity.

Sustained catching-up growth,

$$g_t > g_t^w,$$

raises relative developedness,

$$\rho_t,$$

and thereby supports the material and organizational basis of disciplinary capacity.

Accumulated developmental and command-capacity margins allow the positive-development regime to include temporary periods of relative developmental erosion. The relevant requirement is that the party-state retain sufficient accumulated disciplinary capacity to realize the system-optimal disciplinary level and the bounded adaptive fluctuations around it.

This accumulated command-capacity margin is formalized below as the dynamic command-capacity reserve

$$M_t^P.$$

3.6 System-Optimal Discipline under Positive-Development Capacity

The preceding results establish two components of the developmental problem. First, under the positive-development regime, developmental alignment places aggregate developmental effort at its maximum accessible role-bound level,

$$X_t = \bar{X}_t,$$

so that

$$\Gamma(\bar{X}_t, g_t^{\max}) = g_t^{\max}.$$

Second, discipline reduces harm from residual deviation, but discipline itself imposes a systemic cost. The disciplinary problem is therefore not the maximization of discipline as such. It is the calibration of discipline at the level that maximizes the calibrated development attainable from maximum accessible developmental effort.

For fixed period-t conditions, define the *full-effort calibrated developmental function* by

$$G_t(P) \equiv g_t(\bar{X}_t, P).$$

Using

$$g_t(X, P) = \Gamma(X, g_t^{\max}) - E_t(P) - K(P)$$

and

$$\Gamma(\bar{X}_t, g_t^{\max}) = g_t^{\max},$$

this becomes

$$G_t(P) = g_t^{\max} - E_t(P) - K(P).$$

Equivalently, define the *internal disciplinary burden*:

$$B_t(P) \equiv E_t(P) + K(P).$$

Then

$$G_t(P) = g_t^{\max} - B_t(P).$$

Thus, at maximum accessible developmental effort, maximizing calibrated development is equivalent to minimizing the combined systemic burden generated by residual deviation and discipline itself.

Assumption 3.22 (Regularity of the Internal Disciplinary Burden)**.** For each period t in the positive-development regime, the function

$$B_t(P) = E_t(P) + K(P)$$

is continuous on

$$\mathbb{R}_+,$$

is strictly convex on its relevant domain, and satisfies

$$\lim_{P\to\infty} B_t(P) = +\infty.$$

Where first-order conditions are used, E_t and K are differentiable with respect to P.

The strict convexity condition means that the reduction of deviation-related harm and the increasing cost of discipline generate a unique developmental calibration rather than an interval of equally optimal disciplinary levels. The coercivity condition excludes an unbounded optimum: discipline can be made increasingly severe, but sufficiently high discipline eventually imposes a burden that prevents ever-higher discipline from remaining developmentally optimal.

Definition 3.23 (System-Optimal Discipline)**.** The *system-optimal disciplinary level* is the unique finite level of discipline that maximizes calibrated development under maximum accessible developmental effort:

$$P_t^\star = \arg\max_{P\geq 0} G_t(P).$$

Equivalently,

$$P_t^\star = \arg\min_{P\geq 0} B_t(P).$$

Under Assumption 3.22, the system-optimal disciplinary level exists, is finite, and is unique.

Assumption 3.24 (Interior System-Optimal Discipline)**.** In the positive-development regime analyzed in this section, the unique system-optimal disciplinary level is interior:

$$P_t^\star > 0.$$

Hence,

$$G_t'(P_t^\star) = 0.$$

Under Assumption 3.24, the first-order condition is

$$G_t'(P_t^\star) = 0,$$

or equivalently,

$$-E_t'(P_t^\star) = K'(P_t^\star).$$

Thus, at the system-optimal disciplinary level, the marginal developmental gain obtained from a further reduction in deviation-related harm is exactly offset by the marginal systemic cost of additional discipline.

The system-optimal level

$$P_t^\star$$

is a period-specific developmental optimum that may vary over historical time as systemic conditions alter the structure of residual deviation, the harm generated by residual deviation, and the cost of discipline. In general,

$$P_{t+1}^\star \neq P_t^\star.$$

The disciplinary mechanism developed below therefore tracks a potentially moving system-optimal disciplinary level.

Assumption 3.25 (Positive-Development Disciplinary Capacity Slack)**.** In the positive-development regime, the disciplinary capacity of the party-state is sufficient to realize the system-optimal disciplinary level with strictly positive capacity slack. There exists

$$\delta_t^P > 0$$

such that

$$\bar{P}_t \geq P_t^\star + \delta_t^P.$$

Equivalently,

$$\bar{P}_t - P_t^\star \geq \delta_t^P > 0.$$

This is a regime assumption rather than a result derived solely from the monotonicity properties of disciplinary capacity. The model already specifies disciplinary capacity as

$$\bar{P}_t = \bar{P}(D_t, \rho_t, S_t),$$

with

$$\frac{\partial \bar{P}}{\partial D_t} > 0, \qquad \frac{\partial \bar{P}}{\partial \rho_t} > 0, \qquad \frac{\partial \bar{P}}{\partial S_t} > 0.$$

These relations establish that developedness, relative developedness, and systemic stability support the party-state's capacity to realize discipline, but they do not by themselves imply

$$\bar{P}_t > P_t^\star.$$

The strict inequality is therefore imposed explicitly for the positive-development regime.

Substantively, the assumption represents a constituted party-state that is still developing and reproducing from a sufficiently strong productive, administrative, organizational, and command base that disciplinary capacity is not the binding constraint on developmental calibration. The system-optimal disciplinary level is finite because excessive discipline is itself costly, while the available command capacity of the positive-development party-state is assumed to stand above the finite level actually required to maximize calibrated development. Positive-development disciplinary capacity is finite and exceeds the disciplinary level required for optimal developmental calibration by a strictly positive margin.

This assumption separates the positive-development regime from the exhausted-development and crisis analysis developed later. In the positive-development regime, sufficient disciplinary capacity is maintained as part of the regime conditions. Under exhausted development, the realized trajectory of developedness, relative developedness, stability, and command capacity may instead produce either

$$\bar{P}_t \geq P_t^{\star}$$

or

$$\bar{P}_t < P_t^{\star}.$$

The attainable calibrated developmental frontier is defined over physically accessible aggregate developmental effort and realizable discipline.

Definition 3.26 (Attainable Calibrated Developmental Frontier)**.** The *attainable calibrated developmental frontier* is the maximum calibrated historical efficiency norm attainable over physically accessible aggregate developmental effort and realizable discipline:

$$g_t^A = \max_{\substack{0 \leq X \leq \bar{X}_t \\ 0 \leq P \leq \bar{P}_t}} g_t(X, P).$$

Proposition 3.27 (Attainment of the Calibrated Developmental Frontier under Positive-Development Capacity)**.** *Under Developmental Alignment, Assumption 3.22, and Assumption 3.25, the attainable calibrated developmental frontier is attained at maximum accessible developmental effort and system-optimal discipline:*

$$g_t^A = g_t(\bar{X}_t, P_t^{\star}).$$

Hence,

$$g_t^A = G_t(P_t^{\star}) = g_t^{\max} - E_t(P_t^{\star}) - K(P_t^{\star}).$$

Proof. By Developmental Alignment, aggregate developmental effort reaches its maximum accessible level,

$$X_t = \bar{X}_t.$$

Since

$$\frac{\partial \Gamma(X, g_t^{\max})}{\partial X} > 0,$$

for every fixed disciplinary level P, calibrated growth is maximized with respect to accessible aggregate developmental effort at

$$X = \bar{X}_t.$$

Therefore,

$$\max_{\substack{0 \leq X \leq \bar{X}_t \\ 0 \leq P \leq \bar{P}_t}} g_t(X, P) = \max_{0 \leq P \leq \bar{P}_t} g_t(\bar{X}_t, P).$$

By Definition 3.23,

$$P_t^\star = \arg\max_{P\geq 0} g_t(\bar{X}_t, P).$$

By Assumption 3.25,

$$P_t^\star < \bar{P}_t,$$

so the unconstrained system-optimal disciplinary level lies strictly inside the realizable disciplinary set. Consequently,

$$\max_{0\leq P\leq \bar{P}_t} g_t(\bar{X}_t, P) = g_t(\bar{X}_t, P_t^\star).$$

Using

$$\Gamma(\bar{X}_t, g_t^{\max}) = g_t^{\max},$$

it follows that

$$g_t^A = g_t^{\max} - E_t(P_t^\star) - K(P_t^\star).$$

□

The positive-development regime is therefore characterized by

$$g_t^A > 0,$$

with

$$g_t^A = g_t(\bar{X}_t, P_t^\star),$$

and with the system-optimal disciplinary level strictly realizable under the maintained capacity slack:

$$P_t^\star < \bar{P}_t.$$

This result separates three objects that must remain analytically distinct. The raw frontier efficiency norm

$$g_t^{\max}$$

describes the historical developmental frontier before internal losses are calibrated. The system-optimal disciplinary level

$$P_t^\star$$

minimizes the combined internal burden of deviation-related harm and disciplinary cost. The attainable calibrated developmental frontier

$$g_t^A$$

is the resulting maximum calibrated growth rate that the party-state can attain under maximum accessible developmental effort and realizable discipline.

In the positive-development regime,

$$P_t^\star$$

is physically realizable. The analysis now turns to individual nomenklatura voice adjustments and to the aggregate disciplinary movement generated by those heterogeneous individual adjustments.

3.7 Individual Voice and Adaptive Disciplinary Adjustment

The preceding subsection defines the system-optimal disciplinary level

$$P_t^\star$$

as the finite disciplinary level that maximizes calibrated development under maximum accessible developmental effort. Under the positive-development capacity assumption,

$$P_t^\star < \bar{P}_t,$$

so the developmental optimum is physically realizable by the party-state. The remaining problem is to specify how heterogeneous nomenklatura agents, acting through individual voice, generate changes in aggregate disciplinary pressure.

Nomenklatura agents may possess heterogeneous individual preferences over discipline because they differ in hierarchical position, residual deviation opportunities, exposure to mistaken or excessive discipline, and other position-specific circumstances. The common element of their decision problem is instead that the developedness and reproduction of the party-state enter positively into their systemic position. Individual voice therefore contains both a common developmental component and heterogeneous non-developmental components.

Definition 3.28 (Admissible Individual Voice)**.** For each nomenklatura agent $i \in \mathcal{N}_t$, let

$$\mathcal{V}_{i,t} = [\underline{v}_{i,t}, \bar{v}_{i,t}] \subset \mathbb{R}$$

denote the compact convex set of admissible voice positions available to agent i at time t. The joint admissible voice space is

$$\mathcal{V}_t = \prod_{i \in \mathcal{N}_t} \mathcal{V}_{i,t}.$$

For any voice profile

$$v_t = (v_{i,t})_{i \in \mathcal{N}_t} \in \mathcal{V}_t,$$

aggregate voice remains

$$V_t(v_t) = \sum_{i \in \mathcal{N}_t} w(h_i) v_{i,t}.$$

Recall that the raw disciplinary level induced by aggregate voice is

$$P_t^\Phi(V) = \Phi(V, D_t, \rho_t),$$

with

$$\frac{\partial \Phi(V, D_t, \rho_t)}{\partial V} > 0.$$

Assumption 3.29 (Voice Reachability of System-Optimal Discipline)**.** For every period t in the positive-development regime, the mapping

$$V \mapsto \Phi(V, D_t, \rho_t)$$

is continuous and strictly increasing on the aggregate voice interval generated by

$$\mathscr{V}_t.$$

The system-optimal disciplinary level

$$P_t^\star$$

lies in the image of this mapping. Hence there exists a unique aggregate voice level

$$V_t^\star$$

such that

$$\Phi(V_t^\star, D_t, \rho_t) = P_t^\star.$$

Definition 3.30 (Target Aggregate Voice)**.** The *target aggregate voice* is the unique aggregate voice level

$$V_t^\star$$

that induces the system-optimal disciplinary level:

$$P_t^\Phi(V_t^\star) = \Phi(V_t^\star, D_t, \rho_t) = P_t^\star.$$

Because aggregate voice is a weighted sum of individual voice positions, multiple individual voice profiles may correspond to the same target aggregate voice.

Definition 3.31 (System-Optimal Voice Manifold)**.** The *system-optimal voice manifold* is the set of admissible individual voice profiles that generate target aggregate voice:

$$\mathscr{M}_t^\star = \left\{ v_t \in \mathscr{V}_t : \sum_{i \in \mathscr{N}_t} w(h_i) v_{i,t} = V_t^\star \right\}.$$

Every profile

$$v_t \in \mathscr{M}_t^\star$$

induces the same raw disciplinary level,

$$P_t^\Phi(V_t(v_t)) = P_t^\star.$$

The system-optimal voice manifold separates the microstructure of individual voice from its aggregate disciplinary consequence. Distinct profiles

$$v_t^a \neq v_t^b$$

may satisfy

$$v_t^a, v_t^b \in \mathscr{M}_t^\star$$

and therefore generate

$$V_t(v_t^a) = V_t(v_t^b) = V_t^\star$$

and

$$P_t^{\Phi}(V_t(v_t^a)) = P_t^{\Phi}(V_t(v_t^b)) = P_t^{\star}.$$

Thus, individual voice positions may be redistributed without changing the aggregate disciplinary level whenever the hierarchical-weighted change in voice sums to zero.

In particular, for a change vector

$$\Delta v_t = (\Delta v_{i,t})_{i \in \mathcal{N}_t},$$

if

$$\sum_{i \in \mathcal{N}_t} w(h_i) \Delta v_{i,t} = 0,$$

then aggregate voice is unchanged:

$$\Delta V_t = 0.$$

Such movements alter the internal configuration of voice while leaving the induced disciplinary level unchanged. Individual voice adjustments whose hierarchical-weighted sum is zero leave aggregate voice, and therefore raw voice-induced discipline, unchanged.

To determine the direction of individual voice adjustment when aggregate discipline does change, define the agent's continuation systemic utility conditional on a candidate disciplinary level P.

Under maximum accessible developmental effort,

$$X_t = \bar{X}_t,$$

next-period developedness conditional on discipline P is

$$D_{t+1}(P) = D_t \left[1 + G_t(P)\right].$$

Corresponding relative developedness is

$$\rho_{t+1}(P) = \frac{D_{t+1}(P)}{D_{t+1}^{w}},$$

and next-period systemic stability is

$$S_{t+1}(P) = H(\rho_{t+1}(P), P).$$

Definition 3.32 (Continuation Systemic Utility under Disciplinary Adjustment)**.** For a system-loyal nomenklatura agent i, define continuation systemic utility conditional on disciplinary level P by

$$J_{i,t}(P) = S_{t+1}(P) Z(D_{t+1}(P), h_i) + r_i(P, D_t, h_i) - m_i(P, h_i) F_i(h_i).$$

The function

$$J_{i,t}(P)$$

does not replace the systemic utility used in the loyalty comparison. It is a forward-looking continuation payoff used to represent how a change in current discipline affects the agent through next-period developedness and stability, residual deviation rent, and disciplinary exposure.

The object

$$J_{i,t}(P)$$

must also be distinguished from the one-period continuation payoff

$$\mathscr{C}_{i,t}^{sys}(a_t).$$

The latter is the systemic payoff evaluated at the full state realized in period $t+1$. By contrast,

$$J_{i,t}(P)$$

is a local counterfactual sensitivity object used to construct the adaptive voice update from the inherited period-t disciplinary state. Its derivative with respect to the realized voice configuration is an adjustment signal used to generate

$$v_{i,t+1};$$

it does not mean that period-t voice adjustment retroactively changes the already realized

$$P_t.$$

Where the relevant functions are differentiable,

$$\frac{dD_{t+1}(P)}{dP} = D_t G_t'(P),$$

and

$$\frac{d\rho_{t+1}(P)}{dP} = \frac{D_t}{D_{t+1}^w} G_t'(P).$$

Differentiating continuation systemic utility with respect to discipline gives

$$\frac{dJ_{i,t}(P)}{dP} = A_{i,t}(P)G_t'(P) + \psi_{i,t}(P),$$

where

$$A_{i,t}(P) = H_\rho(\rho_{t+1}(P), P)\frac{D_t}{D_{t+1}^w} Z(D_{t+1}(P), h_i) + H(\rho_{t+1}(P), P)Z_D(D_{t+1}(P), h_i)D_t,$$

and

$$\psi_{i,t}(P) = H_P(\rho_{t+1}(P), P)Z(D_{t+1}(P), h_i) + \frac{\partial r_i(P, D_t, h_i)}{\partial P} - \frac{\partial m_i(P, h_i)}{\partial P} F_i(h_i).$$

Since

$$H_\rho > 0, \qquad Z > 0, \qquad Z_D > 0, \qquad D_t > 0, \qquad D_{t+1}^w > 0,$$

it follows that

$$A_{i,t}(P) > 0.$$

The decomposition

$$\boxed{\frac{dJ_{i,t}(P)}{dP} = A_{i,t}(P)G_t'(P) + \psi_{i,t}(P)}$$

separates two analytically distinct components of individual disciplinary adjustment.

The first component,

$$A_{i,t}(P)G_t'(P),$$

is the agent's *developmental component.* Because

$$A_{i,t}(P) > 0,$$

its sign is common across system-loyal agents and is determined by the slope of the full-effort calibrated developmental function.

For

$$P < P_t^\star,$$

strict optimality of

$$P_t^\star$$

implies

$$G_t'(P) > 0$$

on the relevant side of the optimum, so the developmental component favors an increase in discipline.

For

$$P > P_t^\star,$$

$$G_t'(P) < 0,$$

so the developmental component favors a decrease in discipline.

At

$$P = P_t^\star,$$

$$G_t'(P_t^\star) = 0,$$

and therefore the developmental component vanishes locally.

The second component,

$$\psi_{i,t}(P),$$

is the agent's *non-developmental residual component.* It includes the direct stability effect of discipline, the effect of discipline on residual deviation rent, and the agent's expected disciplinary exposure. This component need not have the same sign or magnitude across agents.

Consequently, individually preferred disciplinary levels may differ across nomenklatura agents. Individual disciplinary positions may differ even though all system-loyal agents share a positive structural exposure to the developmental consequences represented by

$$A_{i,t}(P) > 0.$$

This distinction is central to the adaptive mechanism. Away from the system-optimal disciplinary level, the common developmental component gives individual adjustment a shared directional force. Near the system-optimal level, where

$$G_t'(P)$$

becomes small, heterogeneous residual components can generate continuing differences in individual voice and continuing micro-level adjustment.

Assumption 3.33 (Local Utility-Improving Voice Adjustment)**.** In the positive-development regime, nomenklatura agents adjust individual voice locally in the direction that improves continuation systemic utility, subject to their admissible voice sets. Let

$$\eta_{i,t} > 0$$

denote agent i's adjustment rate, and let

$$\xi_{i,t}$$

denote a transitory idiosyncratic perturbation to individual voice adjustment. Then

$$v_{i,t+1} = \Pi_{\mathscr{V}_{i,t+1}}\left[v_{i,t} + \eta_{i,t}\frac{\partial J_{i,t}}{\partial v_{i,t}} + \xi_{i,t}\right],$$

where

$$\Pi_{\mathscr{V}_{i,t+1}}$$

denotes projection onto the admissible voice set.

Whenever disciplinary capacity is non-binding at the realized voice-induced disciplinary level,

$$P_t = P_t^{\Phi}(V_t) = \Phi(V_t, D_t, \rho_t),$$

The following derivative is a local sensitivity derivative evaluated at the inherited realized state

$$(V_t, P_t).$$

It supplies the directional signal for the update from

$$v_{i,t}$$

to

$$v_{i,t+1}.$$

It is not a contemporaneous causal claim that the period-t adjustment changes

$$P_t$$

after that disciplinary state has already been realized:

$$\frac{\partial J_{i,t}}{\partial v_{i,t}} = \frac{dJ_{i,t}}{dP_t}\frac{\partial P_t}{\partial V_t}\frac{\partial V_t}{\partial v_{i,t}}.$$

Since

$$\frac{\partial P_t}{\partial V_t} = \Phi_V(V_t, D_t, \rho_t) > 0$$

and

$$\frac{\partial V_t}{\partial v_{i,t}} = w(h_i) > 0,$$

it follows that

$$\frac{\partial J_{i,t}}{\partial v_{i,t}} = w(h_i)\Phi_V(V_t, D_t, \rho_t)\left[A_{i,t}(P_t)G_t'(P_t) + \psi_{i,t}(P_t)\right].$$

Thus the adaptive voice rule can be written as

$$v_{i,t+1} = \Pi_{\mathscr{V}_{i,t+1}}\left[v_{i,t} + \eta_{i,t}w(h_i)\Phi_V(V_t, D_t, \rho_t)\left(A_{i,t}(P_t)G_t'(P_t) + \psi_{i,t}(P_t)\right) + \xi_{i,t}\right].$$

Each period generates a new systemic state, aggregate voice, and set of individual continuation incentives. Individual voice positions are therefore repeatedly recalibrated through the adaptive adjustment process.

The resulting process permits persistent movement at the individual level. Some agents may increase voice while others decrease it. Some changes may offset one another exactly and move the profile along the system-optimal voice manifold without changing aggregate discipline. Other changes alter aggregate voice and therefore move realized discipline.

The next step is to aggregate these heterogeneous individual adjustments and determine whether the common developmental component is strong enough to generate a systematic restoring tendency toward the moving system-optimal disciplinary level

$$P_t^\star.$$

3.8 Dynamic Disciplinary Alignment and Moving-Target Tracking

The preceding subsection establishes the individual adaptive mechanism. Each nomenklatura agent adjusts voice in response to a continuation-utility gradient that can be decomposed into a common developmental component and an agent-specific residual component:

$$\frac{dJ_{i,t}(P)}{dP} = A_{i,t}(P)G_t'(P) + \psi_{i,t}(P),$$

with

$$A_{i,t}(P) > 0.$$

The purpose of the present subsection is to determine what happens when these heterogeneous individual adjustments are aggregated.

The central distinction is between persistent movement in the individual composition of voice and systematic movement in aggregate discipline. Individual voice positions may continue to change while aggregate disciplinary adjustment retains a restoring tendency toward the system-optimal disciplinary level

$$P_t^\star.$$

For notational economy, write

$$w_i \equiv w(h_i).$$

Suppose first that the projection constraints in the individual adaptive rule are locally inactive and that disciplinary capacity is non-binding over the relevant adjustment range. The individual adjustment rule then implies

$$v_{i,t+1} - v_{i,t} = \eta_{i,t} w_i \Phi_V(V_t, D_t, \rho_t) \left[A_{i,t}(P_t) G_t'(P_t) + \psi_{i,t}(P_t) \right] + \xi_{i,t}.$$

Since aggregate voice is

$$V_t = \sum_{i \in \mathcal{N}_t} w_i v_{i,t},$$

the corresponding aggregate voice adjustment is

$$\Delta V_t \equiv V_{t+1} - V_t = \sum_{i \in \mathcal{N}_t} w_i (v_{i,t+1} - v_{i,t}).$$

Substitution of the individual adaptive rule yields

$$\Delta V_t = \Phi_V(V_t, D_t, \rho_t) \left[\sum_{i \in \mathcal{N}_t} \eta_{i,t} w_i^2 A_{i,t}(P_t) \right] G_t'(P_t)$$

$$+ \Phi_V(V_t, D_t, \rho_t) \sum_{i \in \mathcal{N}_t} \eta_{i,t} w_i^2 \psi_{i,t}(P_t) + \sum_{i \in \mathcal{N}_t} w_i \xi_{i,t}.$$

Define the aggregate developmental adjustment coefficient:

$$\mathscr{A}_t(P) = \Phi_V(V_t, D_t, \rho_t) \sum_{i \in \mathcal{N}_t} \eta_{i,t} w_i^2 A_{i,t}(P).$$

Since

$$\Phi_V > 0, \qquad \eta_{i,t} > 0, \qquad w_i > 0, \qquad A_{i,t}(P) > 0,$$

it follows that

$$\mathscr{A}_t(P) > 0.$$

Define the aggregate non-developmental adjustment component:

$$\Psi_t(P) = \Phi_V(V_t, D_t, \rho_t) \sum_{i \in \mathcal{N}_t} \eta_{i,t} w_i^2 \psi_{i,t}(P),$$

and define the aggregate idiosyncratic perturbation:

$$\varepsilon_t = \sum_{i \in \mathcal{N}_t} w_i \xi_{i,t}.$$

The aggregate voice adjustment can therefore be written as

$$\boxed{\Delta V_t = \mathscr{A}_t(P_t) G_t'(P_t) + \Psi_t(P_t) + \varepsilon_t.}$$

This equation is the aggregate adaptive law of disciplinary adjustment. The first term is the systematic developmental force. The second and third terms collect heterogeneous position-specific incentives and transitory individual perturbations.

Private and individual motives persist, while their aggregate force remains bounded relative to the common developmental force sufficiently far from the system-optimal disciplinary level.

Assumption 3.34 (Strong Local Curvature and Bounded Residual Adjustment)**.** For each period t in the positive-development regime, there exists a relevant disciplinary interval

$$\mathcal{I}_t \subseteq [0, \bar{P}_t]$$

containing

$$P_t^\star$$

and constants

$$\mu_t > 0, \qquad \underline{\mathscr{A}}_t > 0, \qquad \overline{Q}_t \geq 0,$$

such that, for every

$$P \in \mathcal{I}_t,$$

the following conditions hold:

$$B_t''(P) \geq \mu_t,$$

$$\mathscr{A}_t(P) \geq \underline{\mathscr{A}}_t,$$

and

$$|\Psi_t(P) + \varepsilon_t| \leq \overline{Q}_t.$$

Since

$$G_t(P) = g_t^{\max} - B_t(P),$$

the curvature condition implies

$$G_t''(P) \leq -\mu_t < 0.$$

Thus calibrated development declines with at least locally bounded curvature as discipline moves away from its unique optimum.

Definition 3.35 (Disciplinary Attraction Radius)**.** The *disciplinary attraction radius* is

$$R_t^P = \frac{\overline{Q}_t}{\underline{\mathscr{A}}_t \mu_t}.$$

The attraction radius measures the maximum distance from the system-optimal disciplinary level within which the aggregate residual adjustment force may be strong enough to offset the systematic developmental restoring force.

Outside this radius, the common developmental force necessarily dominates the bounded aggregate residual component.

Proposition 3.36 (Aggregate Restoring Direction of Adaptive Voice)**.** *Under Assumption 3.34, consider any realized disciplinary level*

$$P_t \in \mathcal{I}_t.$$

If

$$P_t < P_t^\star - R_t^P,$$

then

$$\Delta V_t > 0.$$

If

$$P_t > P_t^\star + R_t^P,$$

then

$$\Delta V_t < 0.$$

Hence, outside the attraction radius, aggregate adaptive voice moves in the direction that returns discipline toward the system-optimal disciplinary level.

Proof. Because

$$G_t'(P_t^\star) = 0$$

and

$$G_t''(P) \le -\mu_t,$$

for

$$P < P_t^\star$$

the mean value theorem implies

$$G_t'(P) \ge \mu_t(P_t^\star - P).$$

If

$$P_t < P_t^\star - R_t^P,$$

then

$$P_t^\star - P_t > R_t^P,$$

so

$$G_t'(P_t) > \mu_t R_t^P.$$

Using the definition

$$R_t^P = \frac{\overline{Q}_t}{\underline{\mathscr{A}}_t \mu_t},$$

obtain

$$G_t'(P_t) > \frac{\overline{Q}_t}{\underline{\mathscr{A}}_t}.$$

Since

$$\mathscr{A}_t(P_t) \ge \underline{\mathscr{A}}_t,$$

it follows that

$$\mathscr{A}_t(P_t) G_t'(P_t) > \overline{Q}_t.$$

By Assumption 3.34,

$$|\Psi_t(P_t) + \varepsilon_t| \le \overline{Q}_t.$$

Therefore,

$$\Delta V_t = \mathscr{A}_t(P_t)G_t'(P_t) + \Psi_t(P_t) + \varepsilon_t > 0.$$

The argument for

$$P_t > P_t^\star + R_t^P$$

is symmetric. Strong concavity implies

$$G_t'(P_t) \le -\mu_t(P_t - P_t^\star),$$

and therefore

$$\mathscr{A}_t(P_t)G_t'(P_t) < -\overline{Q}_t.$$

Since the absolute magnitude of the aggregate residual component is bounded by

$$\overline{Q}_t,$$

it follows that

$$\Delta V_t < 0.$$

□

Because

$$\Phi_V(V_t, D_t, \rho_t) > 0,$$

an increase in aggregate voice raises the voice-induced disciplinary level, while a decrease in aggregate voice lowers it. To isolate this voice-induced movement from simultaneous changes in the state variables

$$D_t \quad \text{and} \quad \rho_t,$$

define the disciplinary level generated by the updated aggregate voice while holding the period-t systemic state fixed:

$$\tilde{P}_{t+1}^{(t)} = \Phi(V_{t+1}, D_t, \rho_t).$$

Then Proposition 3.36 implies:

$$P_t < P_t^\star - R_t^P \quad \Rightarrow \quad \tilde{P}_{t+1}^{(t)} > P_t,$$

and

$$P_t > P_t^\star + R_t^P \quad \Rightarrow \quad \tilde{P}_{t+1}^{(t)} < P_t.$$

Thus, holding the systemic state fixed, adaptive voice has a strict restoring direction outside a bounded neighborhood of the system-optimal disciplinary level.

Definition 3.37 (Dynamic Disciplinary Attraction Band)**.** The *dynamic disciplinary attraction band* is

$$\mathscr{B}_t^\star = \left\{P \in \mathscr{I}_t : |P - P_t^\star| \le R_t^P\right\}.$$

Equivalently, where the full interval lies in the nonnegative disciplinary domain,

$$\mathscr{B}_t^\star = \left[P_t^\star - R_t^P, P_t^\star + R_t^P\right].$$

The attraction band is a dynamically admissible neighborhood in which heterogeneous non-developmental incentives and idiosyncratic perturbations may generate continuing local movement in either direction.

Outside the band, aggregate voice is systematically restored toward the band. Inside the band, individual agents may continue to increase or decrease voice, and the exact individual voice profile may change persistently. Micro-level voice movements may persist while aggregate discipline remains systematically oriented toward the developmental optimum.

Assumption 3.38 (Positive-Development Capacity Coverage of the Attraction Band)**.** In the positive-development regime, the capacity slack introduced in Assumption 3.25 is sufficient to contain the normal disciplinary attraction band:

$$R_t^P \le \delta_t^P.$$

Since

$$\bar{P}_t \ge P_t^\star + \delta_t^P,$$

Assumption 3.38 implies

$$P_t^\star + R_t^P \le \bar{P}_t.$$

Hence the entire upper side of the normal attraction band is realizable under positive-development disciplinary capacity.

The positive-development regime therefore separates normal adaptive fluctuation from capacity failure. Within this regime, the party-state possesses sufficient disciplinary capacity to realize both the system-optimal disciplinary level and the normal adaptive neighborhood around it, so ordinary disciplinary fluctuation occurs within a non-binding capacity range.

3.8.1 Aggregation and the Smoothing of Idiosyncratic Voice Perturbations

The aggregate adaptive law contains the idiosyncratic perturbation

$$\varepsilon_t = \sum_{i\in\mathcal{N}_t} w_i \xi_{i,t}.$$

Heterogeneous and imperfectly aligned individual perturbations may substantially offset one another in the aggregate.

Let

$$W_t = \sum_{i\in\mathcal{N}_t} w_i$$

denote total hierarchical weight, and define normalized hierarchical weights:

$$\omega_{i,t} = \frac{w_i}{W_t}.$$

Then

$$\sum_{i \in \mathcal{N}_t} \omega_{i,t} = 1.$$

Define the relative aggregate idiosyncratic perturbation:

$$\varepsilon_t^w = \frac{\varepsilon_t}{W_t} = \sum_{i \in \mathcal{N}_t} \omega_{i,t} \xi_{i,t}.$$

Assumption 3.39 (Idiosyncratic Perturbation Regularity)**.** Conditional on the period-t information set

$$\mathcal{F}_t,$$

individual perturbations satisfy

$$\mathbb{E}[\xi_{i,t} \mid \mathcal{F}_t] = 0,$$

are pairwise conditionally uncorrelated,

$$\mathrm{Cov}(\xi_{i,t}, \xi_{j,t} \mid \mathcal{F}_t) = 0 \qquad \text{for } i \neq j,$$

and have uniformly bounded conditional variance:

$$\mathrm{Var}(\xi_{i,t} \mid \mathcal{F}_t) \leq \sigma_t^2.$$

Lemma 3.40 (Hierarchical-Weighted Smoothing of Idiosyncratic Perturbations)**.** *Under Assumption 3.39,*

$$\mathbb{E}[\varepsilon_t^w \mid \mathcal{F}_t] = 0,$$

and

$$\mathrm{Var}(\varepsilon_t^w \mid \mathcal{F}_t) \leq \sigma_t^2 \sum_{i \in \mathcal{N}_t} \omega_{i,t}^2.$$

Proof. By linearity of conditional expectation,

$$\mathbb{E}[\varepsilon_t^w \mid \mathcal{F}_t] = \sum_{i \in \mathcal{N}_t} \omega_{i,t} \mathbb{E}[\xi_{i,t} \mid \mathcal{F}_t] = 0.$$

Conditional pairwise uncorrelatedness implies

$$\mathrm{Var}(\varepsilon_t^w \mid \mathcal{F}_t) = \sum_{i \in \mathcal{N}_t} \omega_{i,t}^2 \,\mathrm{Var}(\xi_{i,t} \mid \mathcal{F}_t).$$

Using

$$\mathrm{Var}(\xi_{i,t} \mid \mathcal{F}_t) \leq \sigma_t^2$$

gives

$$\mathrm{Var}(\varepsilon_t^w \mid \mathcal{F}_t) \leq \sigma_t^2 \sum_{i \in \mathcal{N}_t} \omega_{i,t}^2.$$

□

The quantity

$$\sum_{i \in \mathcal{N}_t} \omega_{i,t}^2$$

measures the concentration of hierarchical weight. When hierarchical influence is not concentrated in a very small number of agents, heterogeneous idiosyncratic perturbations are partially smoothed in the aggregate. Systematic non-developmental motives remain represented by the aggregate residual component

$$\Psi_t(P).$$

Purely idiosyncratic and uncorrelated movements are partially smoothed through aggregation and therefore exert a weaker aggregate disciplinary effect than perfectly aligned individual movements.

3.8.2 Tracking a Moving System-Optimal Disciplinary Level

The preceding restoring result is stated relative to the period-t system-optimal disciplinary level

$$P_t^\star.$$

However, the optimum itself may change over historical time:

$$P_{t+1}^\star \neq P_t^\star.$$

The relevant dynamic object is the distance between realized discipline and the moving system-optimal target.

Let

$$e_t = P_t - P_t^\star$$

denote the disciplinary tracking error.

The restoring-direction result determines the sign of aggregate adjustment sufficiently far from the optimum. To obtain a quantitative tracking bound, an additional regularity condition is required to control the magnitude of adjustment and rule out persistent explosive overshooting.

Assumption 3.41 (Local Contractivity and Bounded Target Drift)**.** Within the positive-development regime and the relevant disciplinary domain, the combined one-period adaptive disciplinary adjustment generated by individual voice updates and their aggregation satisfies

$$|P_{t+1} - P_t^\star| \leq (1-\kappa_t)\,|P_t - P_t^\star| + u_t,$$

where

$$0 < \underline{\kappa} \leq \kappa_t \leq 1$$

and

$$0 \leq u_t \leq \bar{u} < \infty.$$

The movement of the system-optimal disciplinary target is bounded:

$$\left|P_{t+1}^\star - P_t^\star\right| \leq \bar{\delta} < \infty.$$

The term

$$u_t$$

collects bounded residual displacement generated by heterogeneous non-developmental motives, idiosyncratic perturbations, discrete adjustment, and other locally non-explosive features of the adaptive process. The parameter

$$\kappa_t$$

measures the effective contraction generated by adaptive disciplinary correction.

Assumption 3.41 does not impose the direction of disciplinary adjustment. That direction is established by Proposition 3.36. The assumption controls the magnitude of the already established adaptive process so that movement toward the target does not become permanently destabilizing through unbounded overshooting.

Proposition 3.42 (Moving-Target Tracking of System-Optimal Discipline)**.** *Under Proposition 3.36 and Assumption 3.41, the disciplinary tracking error satisfies*

$$|e_{t+1}| \le (1-\underline{\kappa})|e_t| + \overline{u} + \overline{\delta}.$$

Consequently,

$$\limsup_{t\to\infty} |P_t - P_t^\star| \le \frac{\overline{u} + \overline{\delta}}{\underline{\kappa}}.$$

Proof. By definition,

$$e_{t+1} = P_{t+1} - P_{t+1}^\star.$$

Add and subtract

$$P_t^\star \ :$$

$$e_{t+1} = (P_{t+1} - P_t^\star) - (P_{t+1}^\star - P_t^\star).$$

Therefore, by the triangle inequality,

$$|e_{t+1}| \le |P_{t+1} - P_t^\star| + |P_{t+1}^\star - P_t^\star|.$$

Using Assumption 3.41,

$$|e_{t+1}| \le (1-\kappa_t)|e_t| + u_t + \overline{\delta}.$$

Since

$$\kappa_t \ge \underline{\kappa}$$

and

$$u_t \le \overline{u},$$

it follows that

$$|e_{t+1}| \le (1-\underline{\kappa})|e_t| + \overline{u} + \overline{\delta}.$$

Iterating the inequality gives

$$|e_t| \le (1-\underline{\kappa})^t|e_0| + (\overline{u} + \overline{\delta})\sum_{j=0}^{t-1}(1-\underline{\kappa})^j.$$

Since

$$0 < \underline{\kappa} \leq 1,$$

the geometric sum converges to

$$\frac{1}{\underline{\kappa}}.$$

Hence

$$\limsup_{t\to\infty} |e_t| \leq \frac{\overline{u} + \overline{\delta}}{\underline{\kappa}}.$$

□

The proposition establishes bounded tracking of a moving disciplinary target. The individual voice profile may continue to change, the system-optimal voice manifold may change with the systemic state, and the target disciplinary level itself may move over time. Nevertheless, provided that adaptive adjustment remains locally contractive and structural movement is bounded, realized discipline remains in a bounded neighborhood of the moving developmental optimum.

The width of this tracking neighborhood increases with the magnitude of residual disturbance,

$$\overline{u},$$

and with the speed of target movement,

$$\overline{\delta},$$

and decreases with the strength of adaptive contraction,

$$\underline{\kappa}.$$

The positive-development disciplinary mechanism is a dynamically aligned process. Heterogeneous nomenklatura agents continuously adjust individual voice, individual configurations may change persistently, and idiosyncratic movements may partially offset one another, while the aggregate disciplinary process remains systematically oriented toward the moving system-optimal disciplinary level.

The remaining question is whether the bounded fluctuations permitted by this dynamic process are small enough that calibrated development remains positive and systemic reproduction remains above the critical stability threshold. That robustness problem is addressed in the following subsection.

3.9 Developmental and Reproductive Robustness under Bounded Disciplinary Fluctuations

The preceding results establish that heterogeneous individual voice adjustments can generate a dynamically aligned aggregate disciplinary process. Individual voice profiles may continue to change while adaptive adjustment generates a restoring tendency toward

$$P_t^\star$$

and, under the maintained tracking conditions, keeps realized discipline within a bounded neighborhood of the potentially moving system-optimal target.

The remaining question is whether calibrated development and systemic reproduction remain robust within this bounded disciplinary neighborhood.

The following results establish the corresponding developmental and reproductive margins. Because

$$P_t^\star$$

maximizes the full-effort calibrated developmental function,

$$G_t(P),$$

small disciplinary deviations around the optimum generate only second-order developmental losses. If the attainable calibrated developmental frontier and the stability of the optimally calibrated system possess sufficient positive margins, bounded adaptive fluctuations remain compatible with both positive calibrated growth and systemic reproduction.

For the local robustness results below, maintain the interior system-optimal case

$$P_t^\star > 0,$$

so that

$$G_t'(P_t^\star) = 0.$$

Definition 3.43 (Effective Disciplinary Fluctuation Radius)**.** An *effective disciplinary fluctuation radius* for period t is a bound

$$\widehat{R}_t^P \geq 0$$

such that the realized dynamically adjusted disciplinary level satisfies

$$|P_t - P_t^\star| \leq \widehat{R}_t^P.$$

The attraction and moving-target tracking results developed in the preceding subsection provide the basis for such a bounded neighborhood. The effective fluctuation radius summarizes the realized or guaranteed tracking bound relevant for developmental and reproductive robustness.

Assumption 3.44 (Positive-Development Dynamic Capacity Coverage)**.** For every period t in the positive-development regime, the effective disciplinary fluctuation radius satisfies

$$\widehat{R}_t^P \leq \delta_t^P,$$

where

$$\delta_t^P$$

is the positive capacity slack introduced in Assumption 3.25.

The corresponding disciplinary neighborhood,

$$\left[P_t^\star - \widehat{R}_t^P, P_t^\star + \widehat{R}_t^P\right],$$

is contained in the relevant regularity interval

$$\mathcal{I}_t$$

and in the nonnegative disciplinary domain.

Since

$$\bar{P}_t \geq P_t^\star + \delta_t^P,$$

Assumption 3.44 implies

$$P_t^\star + \widehat{R}_t^P \leq \bar{P}_t.$$

Thus, throughout the bounded dynamic neighborhood considered below, disciplinary capacity remains non-binding.

This assumption completes the role of positive-development capacity slack in the dynamic model. Realized discipline may fluctuate within the bounded tracking neighborhood around

$$P_t^\star.$$

It requires the party-state to possess sufficient capacity to realize the system-optimal disciplinary level together with the bounded adaptive fluctuations generated around it.

Definition 3.45 (Dynamic Command-Capacity Reserve)**.** The *dynamic command-capacity reserve* is the amount of disciplinary capacity remaining above the upper boundary of the effective normal disciplinary fluctuation neighborhood:

$$M_t^P = \bar{P}_t - \left(P_t^\star + \widehat{R}_t^P\right).$$

The reserve

$$M_t^P$$

measures the party-state's current capacity margin after allowing not only for the system-optimal disciplinary level

$$P_t^\star,$$

but also for the full effective upper fluctuation around that moving optimum.

If

$$M_t^P > 0,$$

the entire effective disciplinary fluctuation neighborhood lies strictly below the party-state's maximum realizable disciplinary capacity.

If

$$M_t^P = 0,$$

the upper boundary of the effective disciplinary fluctuation neighborhood exactly reaches available disciplinary capacity.

If

$$M_t^P < 0,$$

the entire effective fluctuation neighborhood is no longer physically realizable. This condition does not by itself imply that the system-optimal disciplinary level

$$P_t^\star$$

is unattainable. The stronger condition

$$\bar{P}_t < P_t^\star$$

is required for the system-optimal level itself to lie beyond available disciplinary capacity.

The dynamic command-capacity reserve must also be distinguished from the positive-development capacity-slack parameter

$$\delta_t^P.$$

The parameter

$$\delta_t^P$$

appears in the maintained positive-development capacity condition

$$\bar{P}_t \geq P_t^\star + \delta_t^P$$

and provides a guaranteed amount of capacity slack above the system-optimal disciplinary level. By contrast,

$$M_t^P$$

measures the actual remaining capacity above the upper boundary

$$P_t^\star + \widehat{R}_t^P$$

of the effective disciplinary fluctuation neighborhood.

Proposition 3.46 (Nonnegative Dynamic Command-Capacity Reserve)**.** *Under the positive-development disciplinary-capacity slack condition*

$$\bar{P}_t \geq P_t^\star + \delta_t^P$$

and the dynamic capacity-coverage condition

$$\widehat{R}_t^P \leq \delta_t^P,$$

the dynamic command-capacity reserve satisfies

$$M_t^P \geq \delta_t^P - \widehat{R}_t^P \geq 0.$$

Hence the entire effective disciplinary fluctuation neighborhood

$$\left[P_t^\star - \widehat{R}_t^P, P_t^\star + \widehat{R}_t^P\right]$$

is physically realizable under available disciplinary capacity.

Proof. By Definition 3.45,

$$M_t^P = \bar{P}_t - \left(P_t^\star + \widehat{R}_t^P\right).$$

The positive-development disciplinary-capacity slack condition gives

$$\bar{P}_t \geq P_t^\star + \delta_t^P.$$

Therefore,

$$M_t^P \geq P_t^\star + \delta_t^P - P_t^\star - \widehat{R}_t^P = \delta_t^P - \widehat{R}_t^P.$$

By the dynamic capacity-coverage condition,

$$\widehat{R}_t^P \leq \delta_t^P,$$

so

$$\delta_t^P - \widehat{R}_t^P \geq 0.$$

Hence,

$$M_t^P \geq 0.$$

It follows that

$$P_t^\star + \widehat{R}_t^P \leq \bar{P}_t,$$

so every realized disciplinary level satisfying

$$|P_t - P_t^\star| \leq \widehat{R}_t^P$$

also satisfies

$$P_t \leq \bar{P}_t.$$

□

An inherited positive dynamic command-capacity reserve allows the party-state to absorb an initial loss of catching-up development without immediate systemic destabilization.

Suppose realized calibrated growth remains positive but falls below the growth rate of the advanced world developedness frontier:

$$0 < g_t < g_t^w.$$

Then,

$$D_{t+1} > D_t,$$

while Proposition 2.12 implies

$$\rho_{t+1} < \rho_t.$$

The party-state is therefore undergoing relative developmental erosion while continuing to develop in absolute terms.

Because disciplinary capacity depends positively on systemic developedness, relative developedness, and stability,

$$\bar{P}_t = \bar{P}(D_t, \rho_t, S_t),$$

relative developmental erosion may place downward pressure on the party-state's available command capacity and on its accumulated capacity reserve.

However, no monotonic change in

$$M_t^P$$

is inferred from a one-period decline in

$$\rho_t$$

alone. The dynamic reserve also depends on the evolution of

$$D_t, \qquad S_t, \qquad P_t^\star, \qquad \widehat{R}_t^P,$$

and these objects may move simultaneously.

The relevant positive-development condition is therefore not that relative developedness must rise in every period. The relevant capacity condition is that the accumulated reserve remain sufficient:

$$M_t^P \geq 0.$$

As long as

$$M_t^P \geq 0,$$

the entire effective disciplinary fluctuation neighborhood remains physically realizable even if

$$\rho_t$$

has begun to decline relative to the advancing world frontier.

Thus, a party-state that accumulated developedness and command capacity during an earlier catching-up phase may continue to sustain positive development and systemic reproduction for some time after its realized growth rate falls below

$$g_t^w.$$

The exhaustion of catching-up momentum and the exhaustion of command capacity are therefore distinct processes.

Assumption 3.47 (Bounded Local Curvature of the Developmental Function)**.** On the effective disciplinary fluctuation neighborhood,

$$\left[P_t^\star - \widehat{R}_t^P, P_t^\star + \widehat{R}_t^P\right],$$

the internal disciplinary burden is twice continuously differentiable and satisfies

$$0 < \mu_t \leq B_t''(P) \leq L_t < \infty.$$

Equivalently,

$$-L_t \leq G_t''(P) \leq -\mu_t < 0.$$

The lower curvature bound preserves strict local optimality. The upper curvature bound limits the rate at which calibrated development deteriorates as realized discipline moves away from the optimum.

Lemma 3.48 (Second-Order Developmental Loss around System-Optimal Discipline)**.** *Under Assumption 3.47, for every realized disciplinary level satisfying*

$$|P_t - P_t^\star| \le \widehat{R}_t^P,$$

the developmental loss relative to system-optimal discipline satisfies

$$\frac{\mu_t}{2}(P_t - P_t^\star)^2 \le G_t(P_t^\star) - G_t(P_t) \le \frac{L_t}{2}(P_t - P_t^\star)^2.$$

Consequently,

$$0 \le G_t(P_t^\star) - G_t(P_t) \le \frac{L_t}{2}(\widehat{R}_t^P)^2.$$

Proof. Since

$$P_t^\star$$

is an interior system-optimal disciplinary level,

$$G_t'(P_t^\star) = 0.$$

By Taylor's theorem, for some point

$$\tilde{P}_t$$

between

$$P_t$$

and

$$P_t^\star,$$

$$G_t(P_t) = G_t(P_t^\star) + \frac{1}{2}G_t''(\tilde{P}_t)(P_t - P_t^\star)^2.$$

Therefore,

$$G_t(P_t^\star) - G_t(P_t) = -\frac{1}{2}G_t''(\tilde{P}_t)(P_t - P_t^\star)^2.$$

Using

$$-L_t \le G_t''(\tilde{P}_t) \le -\mu_t,$$

obtain

$$\frac{\mu_t}{2}(P_t - P_t^\star)^2 \le G_t(P_t^\star) - G_t(P_t) \le \frac{L_t}{2}(P_t - P_t^\star)^2.$$

Finally,

$$|P_t - P_t^\star| \le \widehat{R}_t^P$$

implies

$$G_t(P_t^\star) - G_t(P_t) \le \frac{L_t}{2}(\widehat{R}_t^P)^2.$$

□

The result is important because the first-order developmental loss vanishes at the optimum. When aggregate discipline remains close to

$$P_t^\star,$$

the developmental loss generated by persistent micro-level voice adjustment is second-order. The developmental cost of bounded disciplinary fluctuation is locally quadratic in the distance from the system-optimal level.

Recall that, under positive-development disciplinary capacity,

$$g_t^A = G_t(P_t^\star) > 0.$$

Proposition 3.49 (Robustness of Positive Calibrated Development)**.** *Suppose*

$$|P_t - P_t^\star| \le \widehat{R}_t^P.$$

Under Assumption 3.47, if

$$g_t^A > \frac{L_t}{2}(\widehat{R}_t^P)^2,$$

then realized calibrated growth remains strictly positive:

$$g_t(\bar{X}_t, P_t) > 0.$$

Proof. By Lemma 3.48,

$$G_t(P_t) \ge G_t(P_t^\star) - \frac{L_t}{2}(\widehat{R}_t^P)^2.$$

Since

$$G_t(P_t^\star) = g_t^A,$$

it follows that

$$G_t(P_t) \ge g_t^A - \frac{L_t}{2}(\widehat{R}_t^P)^2.$$

If

$$g_t^A > \frac{L_t}{2}(\widehat{R}_t^P)^2,$$

then

$$G_t(P_t) > 0.$$

Since

$$G_t(P_t) = g_t(\bar{X}_t, P_t),$$

realized calibrated growth is strictly positive. □

The proposition identifies a developmental robustness margin. Positive attainable development remains robust to ordinary adaptive disciplinary fluctuation whenever the realized disciplinary path remains within the established developmental robustness margin. If the optimal calibrated growth rate exceeds the maximum second-order loss generated inside the effective tracking neighborhood, realized development remains positive throughout that neighborhood.

Positive growth alone, however, is not the full reproduction condition. The party-state reproduces only if

$$S_{t+1} \geq S_c.$$

It is therefore necessary to establish that bounded disciplinary fluctuations also preserve sufficient systemic stability.

Define the optimally calibrated next-period developedness:

$$D^{\star}_{t+1} = D_t(1 + g^A_t),$$

the corresponding relative developedness:

$$\rho^{\star}_{t+1} = \frac{D^{\star}_{t+1}}{D^w_{t+1}},$$

and optimally calibrated next-period systemic stability:

$$S^{\star}_{t+1} = H(\rho^{\star}_{t+1}, P^{\star}_t).$$

Definition 3.50 (Optimal Reproduction Margin)**.** The *optimal reproduction margin* is

$$\Delta^S_t = S^{\star}_{t+1} - S_c.$$

A strictly positive optimal reproduction margin,

$$\Delta^S_t > 0,$$

means that under maximum accessible developmental effort and system-optimal discipline, the party-state reproduces with positive stability slack above the critical threshold.

Assumption 3.51 (Local Stability Sensitivity Bounds)**.** On the state-discipline neighborhood generated by

$$|P - P^{\star}_t| \leq \widehat{R}^P_t,$$

the systemic stability function is continuously differentiable and satisfies finite local derivative bounds:

$$0 < H_\rho \leq L^H_{\rho,t} < \infty$$

and

$$0 < H_P \leq L^H_{P,t} < \infty.$$

For realized discipline

$$P_t,$$

next-period developedness is

$$D_{t+1}(P_t) = D_t[1 + G_t(P_t)],$$

and relative developedness is

$$\rho_{t+1}(P_t) = \frac{D_t[1 + G_t(P_t)]}{D^w_{t+1}}.$$

The difference in relative developedness between optimal and realized disciplinary calibration is therefore

$$\rho^\star_{t+1} - \rho_{t+1}(P_t) = \frac{D_t}{D^w_{t+1}} \left[G_t(P^\star_t) - G_t(P_t)\right].$$

Since

$$P^\star_t$$

maximizes

$$G_t,$$

this difference is nonnegative.

By Lemma 3.48,

$$0 \le \rho^\star_{t+1} - \rho_{t+1}(P_t) \le \frac{D_t}{D^w_{t+1}} \frac{L_t}{2} (\widehat{R}^P_t)^2.$$

Proposition 3.52 (Robustness of Systemic Reproduction)**.** *Suppose*

$$|P_t - P^\star_t| \le \widehat{R}^P_t.$$

Under Assumptions 3.47 and 3.51, if

$$\Delta^S_t > L^H_{P,t}\widehat{R}^P_t + L^H_{\rho,t}\frac{D_t}{D^w_{t+1}} \frac{L_t}{2} (\widehat{R}^P_t)^2,$$

then

$$S_{t+1} \ge S_c.$$

Proof. The optimally calibrated stability level is

$$S^\star_{t+1} = H(\rho^\star_{t+1}, P^\star_t),$$

while realized next-period stability is

$$S_{t+1} = H(\rho_{t+1}(P_t), P_t).$$

Using the local derivative bounds,

$$S^\star_{t+1} - S_{t+1} \le L^H_{\rho,t} \left|\rho^\star_{t+1} - \rho_{t+1}(P_t)\right| + L^H_{P,t}|P^\star_t - P_t|.$$

By the effective fluctuation bound,

$$|P^\star_t - P_t| \le \widehat{R}^P_t,$$

and by the second-order developmental-loss bound,

$$\left|\rho^\star_{t+1} - \rho_{t+1}(P_t)\right| \le \frac{D_t}{D^w_{t+1}} \frac{L_t}{2} (\widehat{R}^P_t)^2.$$

Therefore,

$$S^{\star}_{t+1} - S_{t+1} \leq L^{H}_{P,t}\widehat{R}^{P}_{t} + L^{H}_{\rho,t}\frac{D_t}{D^{w}_{t+1}}\frac{L_t}{2}(\widehat{R}^{P}_{t})^2.$$

Since

$$S^{\star}_{t+1} = S_c + \Delta^{S}_{t},$$

the maintained inequality

$$\Delta^{S}_{t} > L^{H}_{P,t}\widehat{R}^{P}_{t} + L^{H}_{\rho,t}\frac{D_t}{D^{w}_{t+1}}\frac{L_t}{2}(\widehat{R}^{P}_{t})^2$$

implies

$$S_{t+1} > S_c.$$

Hence,

$$S_{t+1} \geq S_c.$$

□

The reproduction bound is deliberately conservative. A disciplinary deviation above

$$P^{\star}_{t}$$

may directly raise stability through

$$H_P > 0$$

even while lowering calibrated development through excessive disciplinary cost. The bound is conservative because it treats both direct disciplinary displacement and indirect developmental loss as potential reductions in the reproduction margin. It treats the absolute direct disciplinary displacement and the indirect developmental loss as potential reductions relative to the optimally calibrated reproduction margin.

Proposition 3.53 (Dynamic Reproductive-Growth Robustness). *Under the positive-development regime, suppose that:*

$$|P_t - P^{\star}_{t}| \leq \widehat{R}^{P}_{t} \leq \delta^{P}_{t},$$

$$g^{A}_{t} > \frac{L_t}{2}(\widehat{R}^{P}_{t})^2,$$

and

$$\Delta^{S}_{t} > L^{H}_{P,t}\widehat{R}^{P}_{t} + L^{H}_{\rho,t}\frac{D_t}{D^{w}_{t+1}}\frac{L_t}{2}(\widehat{R}^{P}_{t})^2.$$

Then the dynamically adjusted disciplinary path remains simultaneously:

$$P_t \leq \bar{P}_t,$$

$$g_t(\bar{X}_t, P_t) > 0,$$

and

$$S_{t+1} \geq S_c.$$

Thus bounded adaptive disciplinary fluctuation is compatible with continued positive calibrated development and systemic reproduction.

Proof. By Assumption 3.44,

$$\widehat{R}_t^P \leq \delta_t^P.$$

Together with

$$\bar{P}_t \geq P_t^\star + \delta_t^P,$$

this implies that the upper boundary of the realized fluctuation neighborhood satisfies

$$P_t^\star + \widehat{R}_t^P \leq \bar{P}_t,$$

and therefore realized discipline remains within disciplinary capacity.

By Proposition 3.49, the developmental-margin condition implies

$$g_t(\bar{X}_t, P_t) > 0.$$

By Proposition 3.52, the reproduction-margin condition implies

$$S_{t+1} \geq S_c.$$

Hence the dynamically adjusted disciplinary path remains capacity-feasible, developmentally positive, and reproduction-preserving. □

The resulting positive-development configuration is dynamically aligned. Individual nomenklatura agents may continue to change voice, the distribution of individual voice across the hierarchy may move persistently, and the system-optimal voice manifold and disciplinary level may shift as systemic conditions change.

Nevertheless, the aggregate process remains organized around a moving developmental optimum. Sufficiently large deviations from that optimum generate a restoring developmental force. Heterogeneous individual movements can persist within a bounded neighborhood. Because developmental losses around the optimum are second-order, sufficiently small bounded fluctuations leave calibrated development positive. Where the optimally calibrated system also possesses sufficient stability slack, the same bounded fluctuations remain compatible with reproduction above the critical threshold.

The positive-development regime is sustained by a dynamically aligned disciplinary process whose realized fluctuations remain within the developmental, reproductive, and capacity margins of the constituted party-state.

Under these conditions, the nomenklatura may continuously recalibrate individual voice while the party-state continues to reproduce and develop. The disciplinary mechanism is dynamically variable at the micro level but robust at the systemic level.

3.10 Targeted Blocking of Outlier Collapse Conversion

Proposition 3.7 establishes that post-collapse market conversion is not a generalizable class-wide alternative for the nomenklatura. Nevertheless, isolated agents may possess positive collapse-contingent market exposure before the disciplinary regime acts on that exposure:

$$A_t = \left\{ i \in \mathcal{N}_t \,:\, O_i^C > 0 \right\}.$$

Because the outlier set is non-generalizable, such agents do not define the class-wide position of the nomenklatura. Some outliers may nevertheless be strategically relevant for reproduction and therefore require explicit disciplinary blocking.

Let

$$\chi_i(P, h_i) \geq 0$$

denote the targeted conversion-blocking cost imposed on agent i by disciplinary level P. The effective collapse-contingent market exposure of agent i under discipline P is

$$\widetilde{O}_i^C(P) = O_i^C - \chi_i(P, h_i).$$

Definition 3.54 (Strategically Relevant Outlier)**.** An outlier agent

$$i \in A_t$$

is *strategically relevant* if leaving the agent's positive effective collapse-contingent exposure unblocked creates a potential breach channel that can become reproduction-destroying if the breach becomes effective.

Let

$$A_t^B \subseteq A_t$$

denote the set of strategically relevant outliers.

For every

$$i \in A_t^B,$$

unblocked effective collapse-contingent exposure creates a potential threat to systemic reproduction. Unblocking by itself, however, does not imply that current systemic reproduction has already failed.

Assumption 3.55 (Monotonicity of Targeted Blocking)**.** For every strategically relevant outlier

$$i \in A_t^B,$$

the mapping

$$P \mapsto \chi_i(P, h_i)$$

is continuous and strictly increasing:

$$\frac{\partial \chi_i(P, h_i)}{\partial P} > 0.$$

Higher realized discipline therefore increases the blocking cost imposed on strategically relevant outlier conversion, while lower realized discipline weakens targeted blocking.

Definition 3.56 (Blocking Threshold)**.** The *blocking threshold* is the minimum disciplinary level required to block the positive collapse-contingent exposure of every strategically relevant outlier:

$$P_t^B = \begin{cases} \min\left\{P \geq 0 \;:\; \chi_i(P, h_i) \geq O_i^C \text{ for every } i \in A_t^B\right\}, & \text{if this set is nonempty,} \\ +\infty, & \text{if this set is empty.} \end{cases}$$

Under Assumption 3.55, whenever

$$P_t^B < +\infty,$$

every disciplinary level satisfying

$$P \geq P_t^B$$

blocks every strategically relevant outlier:

$$\chi_i(P, h_i) \geq O_i^C \qquad \text{for every } i \in A_t^B.$$

Equivalently,

$$\widetilde{O}_i^C(P) \leq 0 \qquad \text{for every } i \in A_t^B.$$

Realized discipline may fluctuate within the effective bounded neighborhood

$$|P_t - P_t^\star| \leq \widehat{R}_t^P.$$

Strategic blocking in the positive-development regime is therefore evaluated at the lower boundary of this disciplinary fluctuation neighborhood:

$$P_t^\star - \widehat{R}_t^P.$$

Assumption 3.57 (Positive-Development Robust Strategic Blocking)**.** In the positive-development regime, the lower boundary of the effective disciplinary fluctuation neighborhood is at least as high as the blocking threshold:

$$P_t^\star - \widehat{R}_t^P \geq P_t^B.$$

This is a maintained regime condition for the positive-development reproductive-growth configuration. It states that the normal endogenous fluctuations generated by adaptive voice remain entirely above the disciplinary level required for full strategic-outlier blocking.

Proposition 3.58 (Robust Blocking of Strategically Relevant Outliers)**.** *Suppose*

$$|P_t - P_t^\star| \leq \widehat{R}_t^P$$

and Assumption 3.57 holds. Then realized discipline satisfies

$$P_t \geq P_t^B,$$

and every strategically relevant outlier is blocked:

$$\widetilde{O}_i^C(P_t) \leq 0 \qquad \text{for every } i \in A_t^B.$$

Proof. The effective disciplinary fluctuation bound implies

$$P_t \geq P_t^\star - \widehat{R}_t^P.$$

By Assumption 3.57,

$$P_t^\star - \widehat{R}_t^P \geq P_t^B.$$

Therefore,

$$P_t \geq P_t^B.$$

By Definition 3.56 and Assumption 3.55, any disciplinary level satisfying

$$P_t \geq P_t^B$$

satisfies

$$\chi_i(P_t, h_i) \geq O_i^C$$

for every

$$i \in A_t^B.$$

Hence,

$$\widetilde{O}_i^C(P_t) = O_i^C - \chi_i(P_t, h_i) \leq 0$$

for every strategically relevant outlier. □

Strategic blocking remains intact throughout the effective disciplinary fluctuation neighborhood because its lower boundary lies above the threshold required to contain strategically relevant outlier exposure.

The relationship among the relevant positive-development margins is therefore:

$$P_t^B \leq P_t^\star - \widehat{R}_t^P \leq P_t \leq P_t^\star + \widehat{R}_t^P \leq \bar{P}_t.$$

The lower inequality guarantees strategic-outlier blocking, while the upper inequality follows from positive-development dynamic capacity coverage and guarantees that normal disciplinary fluctuation remains physically realizable.

Outside the positive-development capacity and robustness conditions, realized discipline may fall below

$$P_t^B,$$

allowing strategically relevant outlier exposure to become unblocked. The resulting breach channel becomes collapse-relevant when its systemic consequences contribute to

$$S_{t+1} < S_c.$$

3.11 Loyalty Inequality and the Dynamic Reproductive-Growth Regime

The loyalty condition is comparative. Remaining inside and reproducing the party-state need not yield a payoff that is positive relative to an abstract zero benchmark. Nor is the collapse-contingent alternative treated as riskless. The relevant comparison is between two risk-adjusted payoff conditions:

systemic utility inside the reproduced party-state, which already internalizes expected disciplinary exposure, and effective collapse-contingent market exposure, which incorporates the success probability and downside of post-collapse market conversion.

Both sides of the loyalty comparison are evaluated at the actually realized disciplinary level

$$P_t.$$

For notational clarity, write systemic utility conditional on realized discipline as

$$U_{i,t}^{sys}(P_t) = S_t Z(D_t, h_i) + r_i(P_t, D_t, h_i) - m_i(P_t, h_i) F_i(h_i).$$

The effective collapse-contingent market exposure under the same realized disciplinary level is

$$\widetilde{O}_i^C(P_t) = O_i^C - \chi_i(P_t, h_i).$$

The individual loyalty inequality is therefore

$$\boxed{U_{i,t}^{sys}(P_t) \geq \widetilde{O}_i^C(P_t).}$$

The two probability objects must not be treated as probabilities of the same institutional loss. The term

$$m_i(P_t, h_i)$$

belongs to the intra-system erroneous-or-excessive disciplinary-loss channel. The term

$$1 - \pi_i^C$$

is the probability of unsuccessful market conversion conditional on a collapse branch in which the inherited nomenklatura position has already ceased to be reproduced.

For the institutional comparison, Proposition 2.29 supplies the relevant class-level asymmetry:

$$\bar{d}_t^{D,N} \leq \varepsilon_D^N \ll 1, \qquad \bar{d}_t^{D,w} \leq \varepsilon_D^w \ll 1,$$

inside the reproduced command body, whereas under collapse,

$$\bar{\ell}_{t+1}^{C,N} = \bar{\ell}_{t+1}^{C,w} = 1.$$

The loyalty inequality continues to compare complete individual payoff objects because payoff magnitudes and market-conversion prospects may differ across agents. The structural incidence result therefore supports, but does not replace, the individual loyalty comparison.

The inequality above is the realized-state loyalty comparison at time t. It describes whether the current systemic position dominates effective collapse-contingent exposure under the disciplinary conditions already in force.

A period-t coordinated withdrawal, however, changes the state realized in period $t+1$. The profitability condition relevant to such a forward-looking coalition deviation must therefore use the one-period continuation payoff of the reproduction-preserving branch:

$$\mathscr{C}_{i,t}^{sys}(a_t^R).$$

Definition 3.59 (Decisive-Coalition Stability)**.** Let

$$a_t^R$$

denote the reproduction-preserving period-t benchmark generating the continuation state against which the coordinated deviation is evaluated. A realized reproduction-feasible configuration is *stable against decisive-coalition deviation* at time t if there exists no decisively weighted body

$$B_t \subseteq \mathscr{N}_t$$

for which both of the following conditions hold:

(i) every member strictly reverses the one-period continuation-loyalty comparison,

$$\mathscr{C}_{i,t}^{sys}(a_t^R) < \widetilde{O}_i^C(P_t) \qquad \text{for every } i \in B_t;$$

(ii) a feasible coordinated withdrawal of reproduction-preserving developmental effort and/or discipline-calibrating voice by the agents in B_t, holding the conduct of agents in

$$\mathscr{N}_t \setminus B_t$$

fixed, induces a configuration

$$(\widehat{X}_t, \hat{P}_t)$$

satisfying

$$S_{t+1}(\widehat{X}_t, \hat{P}_t) < S_c.$$

This is a no-profitable-decisive-coalition-deviation criterion. It does not assert that such a coalition must actually form whenever preferences align. It states only that a reproduction-feasible configuration is not coalition-stable if a reproduction-destroying coordinated deviation is both systemically feasible and strictly preferred by every member of a decisively weighted body. The current realized payoff

$$U_{i,t}^{sys}(P_t)$$

is not used as the payoff generated by the contemplated period-t coalition deviation. It remains a realized-state diagnostic.

This comparison is evaluated along the realized dynamic disciplinary path, with

$$P_t$$

remaining within the positive-development robustness conditions established above.

For strategically relevant outliers,

$$i \in A_t^B,$$

Proposition 3.58 implies, throughout the effective disciplinary fluctuation neighborhood,

$$\widetilde{O}_i^C(P_t) \leq 0.$$

Thus, normal adaptive fluctuations around

$$P_t^\star$$

do not reopen the strategically relevant collapse-conversion channel in the positive-development regime.

For the broader weighted body of the nomenklatura, Proposition 3.7 establishes asymptotic non-generalizability of positive collapse-contingent market exposure, while Corollary 3.8 implies that, after the finite threshold

$$T^C,$$

the hierarchical-weighted class-wide exposure is strictly negative. Individual private incentives may remain heterogeneous. The realized-period loyalty condition below remains a distinct condition: it determines whether any decisively weighted body reverses the reproduction-preserving comparison at the actual disciplinary level

$$P_t.$$

Proposition 3.60 (Continuation Loyalty as a Necessary Condition of the Dynamic Reproductive-Growth Regime)**.** *Consider a realized positive-development dynamic reproductive-growth configuration satisfying*

$$X_t = \bar{X}_t,$$

$$|P_t - P_t^\star| \leq \widehat{R}_t^P,$$

$$P_t \leq \bar{P}_t,$$

$$g_t(\bar{X}_t, P_t) > 0,$$

and

$$S_{t+1}(\bar{X}_t, P_t) \geq S_c.$$

Then stability against decisive-coalition deviation requires that there exist no decisively weighted body

$$B_t \subseteq \mathcal{N}_t$$

for which every member strictly reverses the one-period continuation-loyalty comparison:

$$\mathscr{C}_{i,t}^{sys}(a_t^R) < \widetilde{O}_i^C(P_t) \qquad \text{for every } i \in B_t.$$

Equivalently, if such a strict reversal extends across a decisively weighted body whose coordinated withdrawal satisfies the reproduction-destroying condition in Definition 3.59, the realized configuration fails decisive-coalition stability.

The existence of such a preference configuration does not by itself imply that coordination actually occurs or that collapse is immediate.

Proof. The coalition-deviation comparison is forward-looking. Period-t coordinated withdrawal affects whether the nomenklatura position is reproduced in period $t + 1$.

Let

$$a_t^R$$

denote the reproduction-preserving benchmark action profile. Its inside-system payoff for agent i is

$$\mathscr{C}_{i,t}^{sys}(a_t^R).$$

Suppose, contrary to decisive-coalition stability, that there exists a decisively weighted body

$$B_t \subseteq \mathscr{N}_t$$

such that

$$\mathscr{C}_{i,t}^{sys}(a_t^R) < \widetilde{O}_i^C(P_t) \qquad \text{for every } i \in B_t.$$

The strict inequality means that every member of B_t prefers the effective collapse-contingent branch to the reproduced next-period systemic position generated by the reproduction-preserving benchmark.

By Definition 2.33, if the agents in

$$B_t$$

coordinate the relevant reproduction-destroying withdrawal, holding the conduct of the remaining agents fixed, the resulting configuration

$$(\widehat{X}_t, \hat{P}_t)$$

satisfies

$$S_{t+1}(\widehat{X}_t, \hat{P}_t) < S_c.$$

Thus a feasible reproduction-destroying coordinated deviation exists for a body whose members all strictly prefer the collapse-contingent branch to the reproduction-preserving continuation. By Definition 3.59, the realized configuration therefore fails stability against decisive-coalition deviation.

This remains a stability statement rather than an automatic-collapse statement. It does not assert that the coalition necessarily forms or that the destructive action profile is actually implemented.

Hence decisive-coalition stability requires the absence of a decisively weighted body satisfying the strict reversed continuation-loyalty comparison. □

Individual preferences may remain heterogeneous. Isolated agents may prefer alternative conduct, residual deviation may persist, and non-strategically relevant outliers may exist without violating decisive-coalition stability. An individual loyalty reversal is therefore not equivalent to a reproduction-destroying systemic deviation.

The relevant distinction is between individual preference heterogeneity and the existence of a profitable decisive-coalition deviation. Stability fails only when the strict reversed loyalty comparison extends across a decisively weighted body for which the coordinated reproduction-destroying withdrawal identified in Definition 2.33 is feasible. Whether such agents actually coordinate is a separate coalition-formation question that the present model does not need to specify.

Under the positive-development robust-blocking condition, strategically relevant outliers satisfy

$$\widetilde{O}_i^C(P_t) \leq 0,$$

throughout the effective disciplinary fluctuation neighborhood. Together with the non-generalizability of positive collapse conversion for the weighted nomenklatura body, this prevents isolated positive market-conversion opportunities from becoming the stable class-wide basis of reproduction-destroying conduct.

The loyalty result therefore operates on the realized dynamic path:

$$P_t$$

may move around the system-optimal level

$$P_t^\star,$$

but as long as the positive-development dynamic robustness conditions hold, the relevant continuation comparison remains

$$\mathscr{C}_{i,t}^{sys}(a_t^R) \geq \widetilde{O}_i^C(P_t),$$

and a strict reversal extending across a decisively weighted body violates the decisive-coalition stability condition required for a stable reproduction-preserving class position. Such a violation identifies a profitable reproduction-destroying coalition deviation; it does not assert automatic coalition formation.

3.12 Dynamic Systemic Loyalty and the Reproductive-Growth Regime

The preceding results can now be combined into a single positive-development reproduction mechanism.

The nomenklatura's systemic position is tied to the developedness and reproduction of the party-state. Post-collapse market conversion is not a generalizable class-wide alternative. Developmental alignment therefore directs system-loyal nomenklatura agents toward maximum accessible role-bound developmental effort. At that effort level, the disciplinary problem is not the maximization of discipline itself, but the calibration of discipline around the finite system-optimal level

$$P_t^\star$$

that maximizes calibrated development.

Individual nomenklatura agents possess heterogeneous private disciplinary preferences, while their voice adjustments contain both a common developmental component and heterogeneous residual components. Consequently, individual voice profiles may change continuously. Nevertheless, sufficiently large aggregate deviations from

$$P_t^\star$$

generate a common developmental restoring force, while bounded residual motives and idiosyncratic perturbations permit continuing movement inside a limited neighborhood of the moving optimum.

The resulting positive-development configuration is a dynamically aligned reproductive-growth regime in which maximum accessible developmental effort is maintained, realized discipline tracks a potentially moving system-optimal level, normal disciplinary fluctuations remain within the party-state's available capacity, calibrated development remains positive, systemic reproduction remains above the critical threshold, and strategically relevant collapse-conversion channels remain blocked.

Theorem 3.61 (Dynamic Systemic Loyalty and Reproductive-Growth Regime)**.** *Suppose the party-state is in the positive-development regime and the maintained conditions of the preceding results hold.*

In particular:

$$g_t^A > 0,$$

system-loyal nomenklatura agents satisfy Developmental Alignment and therefore choose

$$x_{i,t}^* = \bar{x}_i(g_t^{\max}, h_i),$$

so that

$$X_t = \bar{X}_t;$$

the system-optimal disciplinary level

$$P_t^\star$$

is finite, unique, interior, and attainable under positive-development disciplinary capacity;

individual voice evolves through the adaptive adjustment mechanism;

realized discipline remains within an effective bounded tracking neighborhood,

$$|P_t - P_t^\star| \leq \widehat{R}_t^P;$$

the dynamic capacity condition satisfies

$$\widehat{R}_t^P \leq \delta_t^P;$$

the developmental robustness condition satisfies

$$g_t^A > \frac{L_t}{2}(\widehat{R}_t^P)^2;$$

the reproduction robustness condition satisfies

$$\Delta_t^S > L_{P,t}^H \widehat{R}_t^P + L_{\rho,t}^H \frac{D_t}{D_{t+1}^w} \frac{L_t}{2}(\widehat{R}_t^P)^2;$$

and the robust strategic-blocking condition satisfies

$$P_t^\star - \widehat{R}_t^P \geq P_t^B.$$

Then the realized positive-development path satisfies simultaneously:

$$X_t = \bar{X}_t,$$

$$P_t \leq \bar{P}_t,$$

$$g_t(\bar{X}_t, P_t) > 0,$$

$$S_{t+1}(\bar{X}_t, P_t) \geq S_c,$$

and, for every strategically relevant outlier

$$i \in A_t^B,$$

$$\widetilde{O}_i^C(P_t) \leq 0.$$

Moreover, stability against decisive-coalition deviation requires that no decisively weighted body satisfy the strict reversed one-period continuation-loyalty comparison

$$\mathscr{C}_{i,t}^{sys}(a_t^R) < \widetilde{O}_i^C(P_t).$$

If such a body exists, the configuration fails the decisive-coalition stability condition of Definition 3.59; this does not by itself assert immediate coordination or collapse.

Hence the positive-development configuration is a dynamically aligned systemic-loyalty and reproductive-growth regime.

Proof. Proposition 3.7 establishes asymptotic non-generalizability of a positive class-wide collapse-conversion alternative. Corollary 3.8 strengthens the historical interpretation: for

$$t \geq T^C,$$

$$\bar{O}_t^{C,w} \leq -\frac{L}{2} < 0.$$

Thus, in the mature historical domain identified by the corollary, collapse-contingent market exposure is strictly negative for the nomenklatura as a hierarchical-weighted body. Individual outliers with

$$O_i^C > 0$$

may still exist, but their non-generalizability prevents them from defining the class-wide position.

The theorem distinguishes two loyalty objects. The realized-state comparison,

$$U_{i,t}^{sys}(P_t) \geq \widetilde{O}_i^C(P_t),$$

describes the payoff position already in force at time t. The profitability of period-t reproduction-preserving or reproduction-destroying conduct is governed by the one-period continuation comparison,

$$\mathscr{C}_{i,t}^{sys}(a_t^R) \geq \widetilde{O}_i^C(P_t).$$

Neither condition is identified with the asymptotic class-wide market-exposure result.

By Proposition 3.11, system-loyal nomenklatura agents choose maximum accessible role-bound developmental effort:

$$x^*_{i,t} = \bar{x}_i(g_t^{\max}, h_i).$$

Aggregating across agents gives

$$X_t = \bar{X}_t.$$

At maximum accessible developmental effort, the calibrated developmental problem is summarized by

$$G_t(P) = g_t(\bar{X}_t, P).$$

By Definition 3.23 and the maintained regularity and interiority assumptions, there exists a unique finite system-optimal disciplinary level

$$P_t^\star$$

satisfying

$$G_t'(P_t^\star) = 0.$$

By Assumption 3.25, the party-state possesses positive disciplinary capacity slack around the system-optimal level:

$$\bar{P}_t \geq P_t^\star + \delta_t^P.$$

Individual agents adjust voice according to continuation incentives containing a common developmental component and heterogeneous residual components. Aggregation of those individual adjustments yields the dynamic disciplinary mechanism developed above.

By Proposition 3.36, disciplinary deviations sufficiently far from the system-optimal level generate an aggregate restoring direction toward

$$P_t^\star.$$

By Proposition 3.42, under the maintained contractivity and bounded-target-drift conditions, realized discipline tracks the potentially moving system-optimal disciplinary level within a bounded neighborhood.

Under the effective fluctuation condition,

$$|P_t - P_t^\star| \leq \widehat{R}_t^P,$$

and the dynamic capacity condition,

$$\widehat{R}_t^P \leq \delta_t^P,$$

the realized disciplinary path remains within the party-state's disciplinary capacity:

$$P_t \leq P_t^\star + \widehat{R}_t^P \leq P_t^\star + \delta_t^P \leq \bar{P}_t.$$

By Proposition 3.49, the developmental-margin condition

$$g_t^A > \frac{L_t}{2}(\widehat{R}_t^P)^2$$

implies

$$g_t(\bar{X}_t, P_t) > 0.$$

By Proposition 3.52, the reproduction-margin condition

$$\Delta_t^S > L_{P,t}^H \widehat{R}_t^P + L_{\rho,t}^H \frac{D_t}{D_{t+1}^w} \frac{L_t}{2} (\widehat{R}_t^P)^2$$

implies

$$S_{t+1}(\bar{X}_t, P_t) \geq S_c.$$

Equivalently, Proposition 3.53 establishes jointly that the dynamically adjusted path remains capacity-feasible, developmentally positive, and reproduction-preserving.

By Proposition 3.58, the robust strategic-blocking condition

$$P_t^\star - \widehat{R}_t^P \geq P_t^B$$

implies

$$P_t \geq P_t^B$$

throughout the effective disciplinary fluctuation neighborhood. Therefore,

$$\widetilde{O}_i^C(P_t) \leq 0$$

for every strategically relevant outlier

$$i \in A_t^B.$$

Finally, by Proposition 3.60, a decisively weighted body characterized by a strict reversal

$$U_{i,t}^{sys}(P_t) < \widetilde{O}_i^C(P_t)$$

cannot define a stable reproduction-preserving class position in the realized dynamic reproductive-growth regime.

The resulting configuration therefore combines maximum accessible developmental effort, dynamically adjusted discipline, positive calibrated development, systemic reproduction, strategic-outlier blocking, and class-level systemic loyalty.

Individual voice may continue to change and realized discipline may fluctuate around a moving

$$P_t^\star,$$

while the aggregate dynamic process remains within the capacity, developmental, reproductive, blocking, and loyalty margins established above. □

The resulting positive-development mechanism can be summarized as the following causal sequence:

non-generalizable collapse conversion

$$\Downarrow$$

$$
\begin{gathered}
\text{systemic class interest in party-state reproduction and development} \\
\Downarrow \\
x_{i,t}^{*} = \bar{x}_i(g_t^{\max}, h_i) \quad \Longrightarrow \quad X_t = \bar{X}_t \\
\Downarrow \\
G_t(P) = g_t(\bar{X}_t, P) \quad \Longrightarrow \quad P_t^{\star} = \arg\max_{P \geq 0} G_t(P) \\
\Downarrow \\
\text{heterogeneous adaptive individual voice} \\
\Downarrow \\
\text{aggregate restoring dynamics toward a moving } P_t^{\star} \\
\Downarrow \\
|P_t - P_t^{\star}| \leq \widehat{R}_t^P \\
\Downarrow \\
P_t \leq \bar{P}_t, \qquad g_t(\bar{X}_t, P_t) > 0, \qquad S_{t+1}(\bar{X}_t, P_t) \geq S_c \\
\Downarrow \\
P_t \geq P_t^B \quad \Longrightarrow \quad \widetilde{O}_i^C(P_t) \leq 0 \quad \text{for every } i \in A_t^B \\
\Downarrow \\
\text{dynamic systemic loyalty and reproductive growth.}
\end{gathered}
$$

The central result is a dynamically aligned reproductive-growth regime. The nomenklatura's developmental effort is structurally aligned with the growth of the party-state, while discipline is continuously recalibrated through heterogeneous individual voice. The micro-level configuration may change persistently while the aggregate process remains organized around the moving system-optimal disciplinary level.

Normal disciplinary fluctuations remain compatible with the positive-development regime when they stay within the maintained robustness margins. Under those conditions, the party-state continues to develop, reproduce, retain sufficient command capacity, and block strategically relevant collapse-conversion channels.

Equivalently, under the maintained positive-development capacity conditions, Proposition 3.46 implies

$$M_t^P \geq 0.$$

The positive-development reproductive-growth regime therefore possesses sufficient accumulated command capacity to realize the full effective disciplinary fluctuation neighborhood around the moving system-optimal level.

The positive-development regime may include temporary relative developmental erosion while accumulated developmental and command-capacity margins remain sufficient for continued systemic reproduction.

Under exhausted development, the system-optimal disciplinary level and its normal dynamic neighborhood may cease to be fully attainable under existing disciplinary capacity. Once those guarantees are removed, the nomenklatura may retain an interest in systemic reproduction even when the party-state no longer possesses the command capacity required to realize the disciplinary configuration that reproduction would demand.

4 Exhausted Development, Relative Erosion, and Crisis Bifurcation

The positive-development part of the model analyzed a party-state for which positive calibrated development remains attainable:

$$g_t^A > 0.$$

Within that regime, the system may pass through periods of catching-up development,

$$g_t > g_t^w,$$

periods of relative developmental maintenance,

$$g_t = g_t^w,$$

and even periods of positive absolute development accompanied by relative developmental erosion,

$$0 < g_t < g_t^w.$$

The last case does not by itself constitute exhausted development. A party-state may continue to grow in absolute terms and reproduce systemically while gradually losing ground relative to the advancing world developedness frontier, provided that its accumulated developmental and command-capacity margins remain sufficient.

The present section begins only when positive calibrated development itself is no longer attainable.

Definition 4.1 (Exhausted-Development Regime)**.** The party-state is in the *exhausted-development regime* when the attainable calibrated developmental frontier is non-positive:

$$g_t^A \leq 0.$$

The exhausted-development regime is the developmental crisis regime analyzed in this part of the model. Its defining condition is the exhaustion of attainable positive calibrated development,

$$g_t^A \leq 0,$$

not the immediate failure of systemic reproduction.

Accordingly, exhausted development and collapse are distinct conditions. Once

$$g_t^A \leq 0,$$

the party-state may still remain above the reproduction threshold and therefore continue to reproduce through crisis containment. Collapse occurs only if the realized crisis configuration violates the reproduction condition:

$$S_{t+1} < S_c.$$

Thus,

$$\boxed{\text{exhausted-development crisis} \neq \text{collapse.}}$$

The condition

$$g_t^A \leq 0$$

is stronger than the loss of catching-up development.

A party-state loses catching-up development whenever

$$g_t < g_t^w.$$

It may nevertheless continue to grow absolutely if

$$g_t > 0.$$

By contrast,

$$g_t^A \leq 0$$

means that no physically feasible effort-discipline configuration can generate positive calibrated growth.

Since

$$g_t^A = \max_{\substack{0 \leq X \leq \bar{X}_t \\ 0 \leq P \leq \bar{P}_t}} g_t(X, P),$$

the exhausted-development condition implies

$$g_t(X, P) \leq 0$$

for every physically feasible configuration

$$(X, P).$$

In particular, for the actually realized feasible configuration,

$$g_t \leq 0.$$

Therefore,

$$D_{t+1} = D_t(1 + g_t) \leq D_t.$$

At the same time, the advanced world developedness frontier continues to advance by Assumption 2.11:

$$g_t^w > 0,$$

and therefore

$$D_{t+1}^w > D_t^w.$$

The exhausted-development regime thus creates a structural asymmetry between the party-state and the external developmental frontier. The party-state can no longer generate positive calibrated growth, while the advanced world frontier continues to move forward.

Proposition 4.2 (Necessary Relative Developmental Erosion under Exhausted Development)**.** *Suppose*

$$g_t^A \leq 0$$

and

$$g_t^w > 0.$$

Then every physically feasible realized configuration satisfies

$$\rho_{t+1} < \rho_t.$$

Hence exhausted development necessarily produces relative developmental erosion against an advancing world developedness frontier.

Proof. Under the exhausted-development condition,

$$g_t^A \leq 0.$$

Because

$$g_t^A = \max_{\substack{0 \leq X \leq \bar{X}_t \\ 0 \leq P \leq \bar{P}_t}} g_t(X, P),$$

every physically feasible realized configuration satisfies

$$g_t \leq 0.$$

By Assumption 2.11,

$$g_t^w > 0.$$

Therefore,

$$g_t < g_t^w.$$

By Proposition 2.12,

$$g_t < g_t^w \iff \rho_{t+1} < \rho_t.$$

Hence,

$$\rho_{t+1} < \rho_t.$$

□

This result identifies the first structural consequence of exhausted development.

Before exhaustion, relative developmental erosion may occur temporarily while the party-state continues to grow absolutely:

$$0 < g_t < g_t^w.$$

Such erosion can be absorbed for some time by previously accumulated developedness and command-capacity reserve.

After

$$g_t^A \leq 0,$$

however, the situation changes qualitatively. The party-state can no longer generate positive calibrated growth under any physically feasible effort-discipline configuration, while

$$D_t^w$$

continues to increase.

Relative developmental erosion therefore becomes persistent as long as the exhausted-development condition continues to hold:

$$\rho_{t+1} < \rho_t.$$

The resulting causal sequence begins as:

$$g_t^A \leq 0$$

$$\Downarrow$$

$$g_t \leq 0$$

while

$$g_t^w > 0$$

$$\Downarrow$$

$$\rho_{t+1} < \rho_t.$$

This marks the onset of the crisis mechanism, while the collapse branch remains determined by

$$S_{t+1} < S_c.$$

The party-state may initially retain substantial inherited stocks of developedness, stability, and disciplinary capacity. In particular, the dynamic command-capacity reserve

$$M_t^P$$

may remain nonnegative when

$$g_t^A$$

first becomes non-positive, allowing disciplinary capacity to remain temporarily non-binding.

Persistent relative developmental erosion then creates accumulating structural pressure on the material and organizational conditions from which command capacity is generated.

Recall that:

$$\bar{P}_t = \bar{P}(D_t, \rho_t, S_t),$$

with

$$\bar{P}_D > 0, \qquad \bar{P}_\rho > 0, \qquad \bar{P}_S > 0.$$

Under exhausted development,

$$D_{t+1} \leq D_t,$$

while relative developedness necessarily satisfies

$$\rho_{t+1} < \rho_t.$$

These movements remove the developmental forces that previously supported the expansion or preservation of disciplinary capacity. However, because disciplinary capacity also depends on

$$S_t,$$

and because the required disciplinary target

$$P_t^\star$$

and fluctuation radius

$$\widehat{R}_t^P$$

may themselves change over time, no immediate monotonic conclusion about

$$\bar{P}_t$$

or

$$M_t^P$$

is imposed at this stage.

The next step is therefore to identify the conditions under which persistent relative developmental erosion translates into an actual erosion of the party-state's dynamic command-capacity reserve.

Only after that reserve is sufficiently reduced can disciplinary capacity begin to constrain the adaptive movement of realized discipline around

$$P_t^\star.$$

The crisis mechanism analyzed below is therefore sequential:

$$\text{exhausted calibrated development}$$

$$\Downarrow$$

$$\text{persistent relative developmental erosion}$$

$$\Downarrow$$

possible erosion of accumulated command-capacity reserve

$$\Downarrow$$

possible binding disciplinary capacity

$$\Downarrow$$

crisis containment or collapse.

The following subsections formalize each stage separately.

4.1 Exhausted Development under Maximal Accessible Effort

Under exhausted development, the party-state may continue to receive maximum accessible developmental effort while its attainable calibrated developmental frontier remains non-positive.

Aggregate developmental effort continues to enter calibrated growth through

$$g_t(X, P) = \Gamma(X, g_t^{\max}) - E_t(P) - K(P),$$

with

$$\frac{\partial \Gamma(X, g_t^{\max})}{\partial X} > 0.$$

Hence, for any fixed physically realizable disciplinary level

$$P \in [0, \bar{P}_t],$$

maximum accessible aggregate developmental effort,

$$X = \bar{X}_t,$$

maximizes the frontier-directed contribution to calibrated growth.

At maximum accessible aggregate developmental effort,

$$\Gamma(\bar{X}_t, g_t^{\max}) = g_t^{\max}.$$

Therefore,

$$g_t(\bar{X}_t, P) = g_t^{\max} - E_t(P) - K(P).$$

The exhausted-development condition nevertheless states that positive calibrated growth is no longer attainable anywhere in the physically accessible effort-discipline domain.

Proposition 4.3 (Exhausted Development despite Maximal Accessible Effort)**.** *Suppose the party-state is in the exhausted-development regime:*

$$g_t^A \leq 0,$$

where

$$g_t^A = \max_{\substack{0 \le X \le \bar{X}_t \\ 0 \le P \le \bar{P}_t}} g_t(X, P).$$

Then every physically feasible effort-discipline configuration satisfies

$$g_t(X, P) \le 0.$$

In particular, for every physically realizable disciplinary level

$$P \in [0, \bar{P}_t],$$

maximum accessible aggregate developmental effort satisfies

$$g_t(\bar{X}_t, P) \le 0.$$

Hence even maximum accessible aggregate developmental effort cannot generate positive calibrated development in the exhausted-development regime.

Proof. By definition,

$$g_t^A = \max_{\substack{0 \le X \le \bar{X}_t \\ 0 \le P \le \bar{P}_t}} g_t(X, P).$$

Under exhausted development,

$$g_t^A \le 0.$$

Therefore, if there existed any physically feasible configuration

$$(X', P')$$

satisfying

$$0 \le X' \le \bar{X}_t, \qquad 0 \le P' \le \bar{P}_t,$$

and

$$g_t(X', P') > 0,$$

then the maximum over the physically feasible domain would satisfy

$$g_t^A \ge g_t(X', P') > 0,$$

contradicting

$$g_t^A \le 0.$$

Hence,

$$g_t(X, P) \le 0$$

for every physically feasible configuration.

Since

$$\bar{X}_t$$

belongs to the accessible effort domain, it follows in particular that

$$g_t(\bar{X}_t, P) \le 0$$

for every

$$P \in [0, \bar{P}_t].$$

$\square$

Under

$$g_t^A \le 0,$$

maximum accessible developmental effort is incapable of producing positive calibrated growth at any physically realizable disciplinary level. Once

$$g_t^A \le 0,$$

maximum accessible developmental effort is *incapable* of producing positive calibrated growth at any physically realizable disciplinary level.

Exhausted development can arise while maximum accessible developmental effort continues to be supplied.

The crisis analysis therefore permits system-loyal agents to continue supplying their maximum accessible role-bound effort:

$$x_{i,t}^C = \bar{x}_i(g_t^{\max}, h_i),$$

and therefore

$$X_t^C = \bar{X}_t.$$

Even under maximum accessible developmental effort, Proposition 4.3 gives:

$$g_t(\bar{X}_t, P) \le 0$$

for every physically realizable

$$P.$$

The distinction is between the continued supply of developmental effort and the historical capacity of the system to transform that effort into positive calibrated development.

Before exhaustion, greater aggregate developmental effort raises

$$\Gamma(X_t, g_t^{\max})$$

and contributes to a positive attainable development path.

After

$$g_t^A \le 0,$$

the same accessible effort can still maximize the developmental contribution available to the system, but even that maximal contribution is insufficient to offset the combined internal burden

$$E_t(P) + K(P)$$

at every physically realizable disciplinary level.

Thus the crisis is not generated by an assumed disappearance of nomenklatura effort. It begins with the exhaustion of the system's ability to convert even its maximum accessible developmental effort and best physically available disciplinary calibration into positive growth.

Since every physically feasible realized configuration now satisfies

$$g_t \le 0,$$

while the advanced world developedness frontier continues to satisfy

$$g_t^w > 0,$$

Proposition 4.2 applies:

$$\rho_{t+1} < \rho_t.$$

The resulting structural sequence is therefore:

$$X_t = \bar{X}_t$$

$$\not\Rightarrow$$

$$g_t > 0,$$

and, under exhausted development,

$$g_t \le 0 < g_t^w$$

$$\Longrightarrow$$

$$\rho_{t+1} < \rho_t.$$

The exhausted-development regime is defined by the deeper historical constraint that, even when maximum accessible effort continues to be supplied, the attainable calibrated developmental frontier has become non-positive.

Persistent relative developmental erosion now places the party-state's previously accumulated command-capacity reserve under structural pressure. However, the decline of

$$\rho_t$$

alone is not sufficient to prove that

$$\bar{P}_t$$

or

$$M_t^P$$

must fall in every individual period.

The next step is therefore to formalize the net dynamic conditions under which continued stagnation or decline in

$$D_t,$$

persistent erosion of

$$\rho_t,$$

and the evolution of systemic stability translate into an actual erosion of the dynamic command-capacity reserve

$$M_t^P.$$

4.2 Residual Deviation under Exhausted Development

The exhausted-development regime, defined by $g_t^A \leq 0$, interacts with two actions already present in the model: developmental effort and deviation. Developmental effort is denoted by

$$x_{i,t},$$

and deviation is denoted by

$$e_{i,t}.$$

Developmental effort and deviation are distinct actions in the present model. Developmental effort and deviation are distinct actions: an agent may continue supplying maximum accessible role-bound effort while simultaneously facing stronger residual incentives to deviate. An agent may continue to supply the maximum accessible role-bound developmental effort while also facing a larger residual incentive to deviate when discipline weakens.

Recall that the expected private return from deviation is

$$\mathbb{E}[R_i(e_i, P_t, D_t, h_i)] = (1 - q_i(P_t, h_i))b_i(e_i, D_t, h_i) - q_i(P_t, h_i)f_i(e_i, h_i).$$

Equivalently,

$$\mathbb{E}[R_i(e_i, P_t, D_t, h_i)] = b_i(e_i, D_t, h_i) - q_i(P_t, h_i)\big[b_i(e_i, D_t, h_i) + f_i(e_i, h_i)\big].$$

Because

$$\frac{\partial q_i(P_t, h_i)}{\partial P_t} > 0,$$

a fall in discipline lowers the probability of detection and sanction. Therefore, a fall in discipline raises the expected return from deviation.

Residual deviation rent is

$$r_i(P_t, D_t, h_i) = \max\left\{0, \sup_{e_i>0} \mathbb{E}[R_i(e_i, P_t, D_t, h_i)]\right\}.$$

Hence, when discipline weakens, residual deviation rent weakly rises:

$$P_t \downarrow \quad \Rightarrow \quad r_i(P_t, D_t, h_i) \uparrow .$$

Proposition 4.4 (Residual Rent and Crisis Deepening under Weakened Discipline)**.** *In the exhausted-development regime,*

$$g_t^A \leq 0,$$

if realized discipline weakens from P to P', with:

$$P' \leq P,$$

then residual deviation rent weakly increases:

$$r_i(P', D_t, h_i) \geq r_i(P, D_t, h_i).$$

If aggregate harm from deviation is generated by residual deviation incentives as specified in the existing discipline-deviation result, then:

$$E_t(P') \geq E_t(P).$$

Consequently, holding aggregate developmental effort fixed,

$$g_t(X, P') = \Gamma(X, g_t^{\max}) - E_t(P') - K(P')$$

may be reduced through the increase in deviation-related harm, subject to the simultaneous change in disciplinary cost $K(P)$.

Proof. By the existing monotonicity result:

$$P' \leq P \quad \Rightarrow \quad r_i(P', D_t, h_i) \geq r_i(P, D_t, h_i).$$

By the existing result that discipline weakly reduces aggregate harm from deviation:

$$\frac{\partial E_t(P)}{\partial P} \leq 0.$$

Therefore weaker discipline weakly raises deviation-related harm. □

$$g_t^A \leq 0$$

means positive calibrated growth is unattainable over the joint admissible feasible set. At the same time, if disciplinary capacity weakens and realized discipline falls, then:

$$r_i \uparrow$$

and:

$$E_t \uparrow$$

may further worsen the developmental and stability conditions of the system. The mechanism operates at a fixed level of developmental effort, including the maintained maximum-effort crisis configuration.

4.3 Relative Erosion and the Dynamic Weakening of Command Capacity

Under exhausted development, persistent relative developmental erosion creates a negative developmental contribution to disciplinary capacity, while the total movement of capacity also depends on systemic stability.

Disciplinary capacity is

$$\bar{P}_t = \bar{P}(D_t, \rho_t, S_t),$$

with

$$\bar{P}_D > 0, \qquad \bar{P}_\rho > 0, \qquad \bar{P}_S > 0.$$

Under exhausted development,

$$D_{t+1} \leq D_t,$$

and, because the advanced world developedness frontier continues to advance,

$$\rho_{t+1} < \rho_t.$$

These two movements place downward pressure on

$$\bar{P}_t.$$

However, systemic stability

$$S_t$$

may also change, and a sufficiently large increase in stability could in principle offset part of the developmental pressure on disciplinary capacity.

Moreover, the relevant object for the dynamic disciplinary mechanism is not

$$\bar{P}_t$$

alone. The positive-development mechanism requires capacity sufficient to cover the upper boundary of the effective disciplinary fluctuation neighborhood,

$$P_t^\star + \widehat{R}_t^P.$$

Define

$$U_t^P = P_t^\star + \widehat{R}_t^P.$$

The dynamic command-capacity reserve can therefore be written as

$$M_t^P = \bar{P}_t - U_t^P.$$

The evolution of the reserve depends jointly on the evolution of available disciplinary capacity and on the evolution of the disciplinary requirement represented by

$$U_t^P.$$

Definition 4.5 (Available Disciplinary-Capacity Erosion)**.** The *available disciplinary-capacity erosion* between periods t and $t+1$ is

$$\mathscr{C}_t^P = \bar{P}_t - \bar{P}_{t+1}.$$

Thus,

$$\mathscr{C}_t^P > 0$$

means that maximum realizable disciplinary capacity has fallen,

$$\mathscr{C}_t^P = 0$$

means that it is unchanged, and

$$\mathscr{C}_t^P < 0$$

means that available disciplinary capacity has increased.

Definition 4.6 (Required-Neighborhood Relaxation)**.** The *required-neighborhood relaxation* between periods t and $t+1$ is

$$\mathscr{R}_t^P = U_t^P - U_{t+1}^P = \left(P_t^\star + \widehat{R}_t^P\right) - \left(P_{t+1}^\star + \widehat{R}_{t+1}^P\right).$$

If

$$\mathscr{R}_t^P > 0,$$

the upper boundary of the effective disciplinary neighborhood falls between periods, partially relaxing the amount of capacity required for full dynamic coverage.

If

$$\mathscr{R}_t^P < 0,$$

the required upper disciplinary boundary rises, increasing the capacity burden on the party-state.

The change in the dynamic command-capacity reserve satisfies an exact accounting identity:

$$\begin{aligned} M_{t+1}^P - M_t^P &= \left(\bar{P}_{t+1} - U_{t+1}^P\right) - \left(\bar{P}_t - U_t^P\right) \\ &= -\left(\bar{P}_t - \bar{P}_{t+1}\right) + \left(U_t^P - U_{t+1}^P\right) \\ &= -\mathscr{C}_t^P + \mathscr{R}_t^P. \end{aligned}$$

Therefore,

$$\boxed{M_{t+1}^P < M_t^P \quad \Longleftrightarrow \quad \mathscr{C}_t^P > \mathscr{R}_t^P.}$$

The reserve declines when the erosion of available disciplinary capacity exceeds any contemporaneous relaxation in the upper disciplinary requirement.

This condition also covers the case in which the required disciplinary neighborhood becomes more demanding. If

$$\mathscr{R}_t^P < 0,$$

then an increase in

$$U_t^P$$

reinforces reserve erosion rather than offsetting it.

To connect available capacity erosion to the underlying state variables, consider the transition from

$$(D_t, \rho_t, S_t)$$

to

$$(D_{t+1}, \rho_{t+1}, S_{t+1}).$$

Assume

$$\bar{P}(D, \rho, S)$$

is continuously differentiable on a convex neighborhood containing the transition path. For

$$\lambda \in [0, 1],$$

define

$$z_t(\lambda) = (D_t + \lambda(D_{t+1} - D_t),\ \rho_t + \lambda(\rho_{t+1} - \rho_t),\ S_t + \lambda(S_{t+1} - S_t)).$$

By the fundamental theorem of calculus along this path,

$$\bar{P}_{t+1} - \bar{P}_t = \int_0^1 \Big[\bar{P}_D(z_t(\lambda))(D_{t+1} - D_t) + \bar{P}_\rho(z_t(\lambda))(\rho_{t+1} - \rho_t) + \bar{P}_S(z_t(\lambda))(S_{t+1} - S_t)\Big]\, d\lambda.$$

Define the developmental erosion pressure on disciplinary capacity as

$$\mathscr{D}_t^P = -\int_0^1 \Big[\bar{P}_D(z_t(\lambda))(D_{t+1} - D_t) + \bar{P}_\rho(z_t(\lambda))(\rho_{t+1} - \rho_t)\Big]\, d\lambda.$$

Under exhausted development,

$$D_{t+1} - D_t \leq 0$$

and

$$\rho_{t+1} - \rho_t < 0.$$

Since

$$\bar{P}_D > 0$$

and

$$\bar{P}_\rho > 0,$$

it follows that

$$\mathscr{D}_t^P > 0.$$

Thus exhausted development generates strictly positive developmental erosion pressure on disciplinary capacity through the combined failure of absolute developedness to rise and the necessary decline of relative developedness.

Define the stability contribution to disciplinary capacity as

$$\mathcal{S}_t^P = \int_0^1 \bar{P}_S(z_t(\lambda))\,(S_{t+1} - S_t)\, d\lambda.$$

Then

$$\bar{P}_{t+1} - \bar{P}_t = -\mathscr{D}_t^P + \mathscr{S}_t^P,$$

and therefore

$$\mathscr{C}_t^P = \mathscr{D}_t^P - \mathscr{S}_t^P.$$

If

$$S_{t+1} < S_t,$$

then

$$\mathscr{S}_t^P < 0,$$

so declining stability reinforces the erosion of disciplinary capacity.

If

$$S_{t+1} > S_t,$$

then

$$\mathscr{S}_t^P > 0,$$

so increasing stability partially offsets the developmental erosion pressure.

The reserve change can therefore be written as

$$M_{t+1}^P - M_t^P = -\mathscr{D}_t^P + \mathscr{S}_t^P + \mathscr{R}_t^P.$$

Assumption 4.7 (Net Dynamic Command-Capacity Erosion)**.** Along the persistent exhausted-development path considered in the crisis analysis, the combined developmental erosion pressure exceeds the sum of any stability-based compensation and any relaxation of the required upper disciplinary neighborhood:

$$\mathscr{D}_t^P > \mathscr{S}_t^P + \mathscr{R}_t^P.$$

Along the crisis path analyzed here, the negative command-capacity consequences of non-increasing systemic developedness and declining relative developedness exceed the combined compensation generated by systemic stability and any relaxation of

$$P_t^\star + \widehat{R}_t^P.$$

Proposition 4.8 (Erosion of the Dynamic Command-Capacity Reserve)**.** *Under exhausted development,*

$$g_t^A \le 0,$$

the advancing-world-frontier assumption,

$$g_t^w > 0,$$

and Assumption 4.7, the dynamic command-capacity reserve strictly declines:

$$M_{t+1}^P < M_t^P.$$

Proof. Under exhausted development and an advancing world developedness frontier,

$$D_{t+1} \leq D_t$$

and

$$\rho_{t+1} < \rho_t.$$

Therefore,

$$\mathscr{D}_t^P > 0.$$

From the reserve-change identity,

$$M_{t+1}^P - M_t^P = -\mathscr{D}_t^P + \mathscr{S}_t^P + \mathscr{R}_t^P.$$

By Assumption 4.7,

$$\mathscr{D}_t^P > \mathscr{S}_t^P + \mathscr{R}_t^P.$$

Hence,

$$-\mathscr{D}_t^P + \mathscr{S}_t^P + \mathscr{R}_t^P < 0.$$

Therefore,

$$M_{t+1}^P - M_t^P < 0,$$

which implies

$$M_{t+1}^P < M_t^P.$$

□

The proposition formalizes the gradual exhaustion of inherited command-capacity slack.

At the beginning of exhausted development, the party-state may inherit a substantial positive reserve from its previous developmental trajectory:

$$M_t^P > 0.$$

An inherited positive command-capacity reserve can delay the onset of binding disciplinary capacity after positive calibrated development has been exhausted.

Under persistent net command-capacity erosion, however,

$$M_{t+1}^P < M_t^P.$$

The previously accumulated reserve is progressively consumed.

An inherited positive command-capacity reserve creates a temporal interval between developmental exhaustion and binding disciplinary incapacity.

A party-state may first cease catching up, then continue growing more slowly than the advanced world frontier, then lose the ability to generate positive calibrated growth, and only later exhaust enough of its inherited command-capacity reserve for disciplinary capacity to become binding.

The crisis path is therefore cumulative:

$$g_t^A \leq 0$$

$$\Downarrow$$

$$D_{t+1} \leq D_t, \qquad \rho_{t+1} < \rho_t$$

$$\Downarrow$$

$$\mathscr{D}_t^P > 0$$

$$\Downarrow$$

under the net erosion condition,

$$M_{t+1}^P < M_t^P.$$

The decline of

$$M_t^P$$

does not itself imply that the system-optimal disciplinary level has already become unattainable.

The first boundary reached is the loss of full coverage of the effective disciplinary fluctuation neighborhood:

$$M_t^P < 0 \quad \Longleftrightarrow \quad \bar{P}_t < P_t^\star + \widehat{R}_t^P.$$

Even after this boundary is crossed, it remains possible that

$$\bar{P}_t \geq P_t^\star,$$

so that the exact system-optimal disciplinary level itself remains physically realizable even though the entire normal fluctuation neighborhood no longer does.

A stronger capacity failure occurs only if

$$\bar{P}_t < P_t^\star.$$

4.4 Capacity Boundaries and Constrained Disciplinary Attainability

The erosion of the dynamic command-capacity reserve generates analytically distinct capacity boundaries as available capacity declines relative to the disciplinary requirements of the system.

Recall that the upper boundary of the effective normal disciplinary fluctuation neighborhood is

$$U_t^P = P_t^\star + \widehat{R}_t^P,$$

and that the dynamic command-capacity reserve is

$$M_t^P = \bar{P}_t - U_t^P.$$

The first capacity boundary concerns the feasibility of the entire normal dynamic fluctuation neighborhood.

If

$$M_t^P \geq 0,$$

then

$$\bar{P}_t \geq P_t^\star + \widehat{R}_t^P,$$

and the full effective neighborhood

$$\left[P_t^\star - \widehat{R}_t^P, P_t^\star + \widehat{R}_t^P\right]$$

is physically realizable.

If instead

$$M_t^P < 0,$$

then

$$\bar{P}_t < P_t^\star + \widehat{R}_t^P.$$

The party-state can no longer guarantee physical realization of the entire effective disciplinary fluctuation neighborhood.

This is the loss of full dynamic fluctuation coverage. In the intermediate region

$$P_t^\star \leq \bar{P}_t < P_t^\star + \widehat{R}_t^P,$$

the exact system-optimal disciplinary level

$$P_t^\star$$

remains attainable.

The second capacity boundary is reached only when

$$\bar{P}_t < P_t^\star.$$

At this point, the unconstrained system-optimal disciplinary level lies above the maximum level of discipline the party-state can physically realize.

Definition 4.9 (Attainable Optimal Discipline)**.** The *attainable optimal disciplinary level* is the physically realizable disciplinary level that maximizes calibrated development under maximum accessible aggregate developmental effort:

$$P_t^A = \arg\max_{0 \leq P \leq \bar{P}_t} G_t(P),$$

where

$$G_t(P) = g_t(\bar{X}_t, P).$$

Proposition 4.10 (Attainable Optimal Discipline under a Capacity Constraint)**.** *Under the maintained strict convexity of*

$$B_t(P) = E_t(P) + K(P),$$

and the unique interior unconstrained system-optimal disciplinary level

$$P_t^\star,$$

the attainable optimal disciplinary level is

$$\boxed{P_t^A = \min\left\{P_t^\star, \bar{P}_t\right\}.}$$

Proof. At maximum accessible aggregate developmental effort,

$$G_t(P) = g_t^{\max} - B_t(P).$$

Therefore maximizing

$$G_t(P)$$

is equivalent to minimizing

$$B_t(P).$$

By strict convexity of

$$B_t,$$

the unconstrained minimizer

$$P_t^\star$$

is unique.

If

$$P_t^\star \le \bar{P}_t,$$

then the unconstrained optimum belongs to the physically feasible interval

$$[0, \bar{P}_t],$$

so

$$P_t^A = P_t^\star.$$

If instead

$$P_t^\star > \bar{P}_t,$$

strict convexity and optimality of

$$P_t^\star$$

imply that

$$G_t(P)$$

is increasing on the feasible interval up to

$$\bar{P}_t < P_t^\star.$$

Hence the constrained maximum is attained at the upper feasible boundary:

$$P_t^A = \bar{P}_t.$$

Therefore,

$$P_t^A = \min\left\{P_t^\star, \bar{P}_t\right\}.$$

□

Definition 4.11 (Disciplinary-Capacity Shortfall)**.** The *disciplinary-capacity shortfall* relative to the unconstrained system-optimal disciplinary level is

$$\Delta_t^P = \left(P_t^\star - \bar{P}_t\right)_+ = \max\left\{P_t^\star - \bar{P}_t, 0\right\}.$$

Using Proposition 4.10, the shortfall can equivalently be written as

$$\Delta_t^P = P_t^\star - P_t^A.$$

Hence,

$$\Delta_t^P = 0$$

if and only if

$$\bar{P}_t \geq P_t^\star,$$

whereas

$$\Delta_t^P > 0$$

if and only if

$$\bar{P}_t < P_t^\star.$$

When

$$\Delta_t^P > 0,$$

the exact system-optimal disciplinary level is physically unattainable.

Indeed,

$$\min_{0 \leq P \leq \bar{P}_t} |P - P_t^\star| = \Delta_t^P.$$

Thus,

$$\Delta_t^P$$

is also the minimum unavoidable tracking error created purely by insufficient disciplinary capacity.

This result identifies a fundamental distinction between adaptive disciplinary pressure and command capacity.

Individual and aggregate voice may continue to generate pressure toward

$$P_t^\star.$$

The raw voice-induced disciplinary level remains

$$P_t^\Phi(V_t) = \Phi(V_t, D_t, \rho_t).$$

But realized discipline remains

$$P_t = \min\left\{P_t^\Phi(V_t), \bar{P}_t\right\}.$$

Therefore, if

$$\Delta_t^P > 0,$$

no adaptive voice process can make realized discipline equal to

$$P_t^\star.$$

If aggregate voice is sufficiently strong that

$$P_t^\Phi(V_t) \geq \bar{P}_t,$$

then

$$P_t = \bar{P}_t = P_t^A < P_t^\star,$$

and the realized shortfall from the system-optimal disciplinary level is exactly

$$P_t^\star - P_t = \Delta_t^P.$$

This is a capacity-constrained tracking failure.

Adaptive disciplinary pressure may remain directed toward stronger calibration even when the party-state's command apparatus can no longer transform that pressure into the system-optimal realized level.

The third capacity boundary concerns strategic-outlier blocking.

Recall from Definition 3.56 that

$$P_t^B$$

is the minimum disciplinary level required to block all strategically relevant outliers.

Full strategic blocking is physically possible only if

$$\bar{P}_t \geq P_t^B.$$

If instead

$$\bar{P}_t < P_t^B,$$

then every physically realizable disciplinary level satisfies

$$P_t \leq \bar{P}_t < P_t^B.$$

The party-state then lacks the physical disciplinary capacity required for full strategic-outlier blocking.

The three contemporaneous capacity boundaries are therefore:

$$\boxed{\bar{P}_t < P_t^\star + \widehat{R}_t^P}$$

for loss of full dynamic fluctuation coverage;

$$\boxed{\bar{P}_t < P_t^\star}$$

for loss of attainability of the system-optimal disciplinary level;

and

$$\boxed{\bar{P}_t < P_t^B}$$

for loss of physical capacity for full strategic-outlier blocking.

These boundaries must not be conflated.

It is possible to have

$$\bar{P}_t < P_t^\star + \widehat{R}_t^P$$

while still having

$$\bar{P}_t \geq P_t^\star.$$

In that case, the full normal fluctuation neighborhood is no longer covered, but the exact system-optimal level remains attainable.

It is also possible, depending on the contemporaneous location of

$$P_t^B,$$

for the system-optimal target to become unattainable while strategic blocking remains physically possible.

Whenever the contemporaneous threshold ordering

$$P_t^B \leq P_t^\star \leq P_t^\star + \widehat{R}_t^P$$

holds, the corresponding capacity requirements are nested:

$$\bar{P}_t \geq P_t^\star + \widehat{R}_t^P$$

implies

$$\bar{P}_t \geq P_t^\star,$$

which in turn implies

$$\bar{P}_t \geq P_t^B.$$

The reverse implications do not generally hold.

To describe when these boundaries are first crossed along an exhausted-development path, let

$$t_0$$

denote the initial period of the crisis interval under consideration.

Definition 4.12 (Capacity-Boundary Crossing Times)**.** Define:

$$\tau_R = \inf\left\{t \geq t_0 \ : \ M_t^P < 0\right\},$$

$$\tau_\star = \inf\left\{t \geq t_0 \ : \ \Delta_t^P > 0\right\},$$

and

$$\tau_B = \inf\left\{t \geq t_0 \ : \ \bar{P}_t < P_t^B\right\}.$$

If the relevant set is empty, the corresponding crossing time is defined as

$$+\infty.$$

The crossing time

$$\tau_R$$

marks the first loss of full effective fluctuation coverage.

The crossing time

$$\tau_\star$$

marks the first period in which the unconstrained system-optimal disciplinary level itself becomes physically unattainable.

The crossing time

$$\tau_B$$

marks the first period in which full strategic-outlier blocking becomes physically impossible.

Because the thresholds

$$P_t^\star, \qquad \widehat{R}_t^P, \qquad P_t^B$$

may themselves move over time, the crossing times

$$\tau_R, \qquad \tau_\star, \qquad \tau_B$$

may occur in different temporal orders.

If the threshold ordering

$$P_t^B \le P_t^\star \le P_t^\star + \widehat{R}_t^P$$

is preserved along the relevant crisis path and the available capacity path crosses these moving thresholds without threshold overtaking, then the natural severity ordering is

$$\tau_R \le \tau_\star \le \tau_B.$$

This ordering is conditional rather than definitional.

The central substantive result is independent of the exact temporal ordering.

Once

$$\Delta_t^P > 0,$$

adaptive disciplinary pressure toward

$$P_t^\star$$

can no longer be fully translated into realized discipline.

Once

$$\bar{P}_t < P_t^B,$$

even the disciplinary level required for full strategic-outlier blocking is physically unavailable.

The following subsection examines the strategic consequences of this latter boundary.

4.5 Strategic Outlier Unblocking under Insufficient Command Capacity

Strategically relevant outliers provide one channel through which the erosion of disciplinary capacity may become systemically consequential.

Recall that

$$A_t^B$$

denotes the set of strategically relevant outliers and that their effective collapse-contingent market exposure under disciplinary level

$$P$$

is

$$\widetilde{O}_i^C(P) = O_i^C - \chi_i(P, h_i).$$

Recall also from Definition 3.56 that

$$P_t^B$$

is the minimum disciplinary level required to block all strategically relevant outliers.

Under the maintained monotonicity of targeted blocking,

$$\frac{\partial \chi_i(P, h_i)}{\partial P} > 0,$$

stronger realized discipline increases the blocking term

$$\chi_i(P, h_i)$$

and therefore lowers effective collapse-contingent exposure:

$$\frac{\partial \widetilde{O}_i^C(P)}{\partial P} = -\frac{\partial \chi_i(P, h_i)}{\partial P} < 0.$$

In the positive-development regime, robust strategic blocking was maintained throughout the effective disciplinary fluctuation neighborhood. By Proposition 3.58, the condition

$$P_t^\star - \widehat{R}_t^P \geq P_t^B$$

ensures that every realized disciplinary level inside the effective neighborhood satisfies

$$P_t \geq P_t^B,$$

and therefore

$$\widetilde{O}_i^C(P_t) \leq 0$$

for every

$$i \in A_t^B.$$

The crisis mechanism removes this guarantee.

As the dynamic command-capacity reserve erodes, the party-state may eventually lose the physical capacity required to realize the blocking threshold.

The relevant distinction is between:

$$P_t < P_t^B,$$

which means that the realized disciplinary level is insufficient for full strategic blocking, and

$$\bar{P}_t < P_t^B,$$

which means that full strategic blocking is physically impossible under any realizable disciplinary choice.

Definition 4.13 (Unblocked Outlier Set)**.** The *unblocked outlier set* under disciplinary level P is

$$A_t^U(P) = \left\{ i \in A_t^B \, : \, \widetilde{O}_i^C(P) > 0 \right\}.$$

Equivalently,

$$A_t^U(P) = \left\{ i \in A_t^B \, : \, O_i^C > \chi_i(P, h_i) \right\}.$$

An agent belongs to

$$A_t^U(P)$$

when targeted disciplinary blocking is insufficient to eliminate that strategically relevant outlier's positive collapse-contingent exposure.

An unblocked outlier may still satisfy the loyalty condition

$$U_{i,t}^{sys}(P) \geq \widetilde{O}_i^C(P).$$

Thus,

$$\widetilde{O}_i^C(P) > 0$$

means that a positive collapse-conversion exposure has reappeared after disciplinary blocking, but a reversal of systemic loyalty requires the stronger inequality

$$U_{i,t}^{sys}(P) < \widetilde{O}_i^C(P).$$

The distinction between outlier unblocking and loyalty reversal is essential.

Lemma 4.14 (Discipline below the Blocking Threshold)**.** *Suppose*

$$A_t^B \neq \emptyset.$$

If a physically realized disciplinary level satisfies

$$P < P_t^B,$$

with

$$P_t^B = +\infty$$

allowed, then

$$A_t^U(P) \neq \emptyset.$$

Hence at least one strategically relevant outlier remains unblocked.

Proof. By Definition 3.56,

$$P_t^B$$

is the minimum disciplinary level at which

$$\chi_i(P, h_i) \geq O_i^C$$

holds for every

$$i \in A_t^B,$$

when such a finite disciplinary level exists.

Suppose first that

$$P_t^B < +\infty.$$

If

$$P < P_t^B,$$

then P does not belong to the set of disciplinary levels that block every strategically relevant outlier. Therefore there exists at least one

$$i \in A_t^B$$

such that

$$\chi_i(P, h_i) < O_i^C.$$

Hence

$$\widetilde{O}_i^C(P) = O_i^C - \chi_i(P, h_i) > 0,$$

so

$$i \in A_t^U(P).$$

Therefore,

$$A_t^U(P) \neq \emptyset.$$

If instead

$$P_t^B = +\infty,$$

then by definition no finite disciplinary level blocks every strategically relevant outlier. Hence every finite physically realized disciplinary level

$$P$$

leaves at least one

$$i \in A_t^B$$

with

$$\widetilde{O}_i^C(P) > 0.$$

Therefore again,

$$A_t^U(P) \neq \emptyset.$$

□

The lemma concerns the realized disciplinary level.

It is therefore possible for strategically relevant outliers to become unblocked even when full blocking remains physically possible, if realized discipline happens to satisfy

$$P_t < P_t^B$$

despite

$$\bar{P}_t \geq P_t^B.$$

The stronger crisis condition is a failure of blocking capacity itself.

Corollary 4.15 (Capacity–Blocking Impossibility)**.** *Suppose*

$$A_t^B \neq \emptyset.$$

If

$$\bar{P}_t < P_t^B,$$

then every physically realizable disciplinary level

$$P \in [0, \bar{P}_t]$$

satisfies

$$P < P_t^B.$$

Consequently,

$$A_t^U(P) \neq \emptyset$$

for every physically realizable disciplinary level.

Hence no physically realizable disciplinary configuration can fully block all strategically relevant outliers.

Proof. Let

$$P \in [0, \bar{P}_t]$$

be any physically realizable disciplinary level.

Then

$$P \leq \bar{P}_t.$$

Under

$$\bar{P}_t < P_t^B,$$

it follows that

$$P < P_t^B.$$

By Lemma 4.14, since

$$A_t^B \neq \emptyset,$$

we obtain

$$A_t^U(P) \neq \emptyset.$$

Because the argument applies to every

$$P \in [0, \bar{P}_t],$$

no physically realizable disciplinary level can satisfy the full strategic-blocking requirement. □

Corollary 4.15 identifies a genuine command-capacity failure.
When

$$\bar{P}_t < P_t^B,$$

additional voice in favor of stronger discipline cannot restore full strategic blocking.
Even if aggregate voice generates a raw disciplinary demand satisfying

$$P_t^{\Phi}(V_t) \geq P_t^B,$$

realized discipline remains bounded by

$$P_t = \min\left\{P_t^{\Phi}(V_t), \bar{P}_t\right\} \leq \bar{P}_t < P_t^B.$$

The inability to block strategically relevant outliers arises from insufficient realizable command capacity even when raw disciplinary pressure is sufficiently strong.
At the capacity-boundary crossing time

$$\tau_B,$$

the condition

$$\bar{P}_t < P_t^B$$

is reached for the first time along the crisis interval, provided the corresponding crossing set is nonempty.
Because both

$$\bar{P}_t$$

and

$$P_t^B$$

may continue to move over time, subsequent recrossing of the blocking-capacity boundary remains possible after

$$\tau_B.$$

At any period in which

$$\bar{P}_t < P_t^B,$$

however, full strategic blocking is physically impossible.
This opens a strategic breach channel:

$$\bar{P}_t < P_t^B$$

$$\Longrightarrow$$

$$P_t < P_t^B \quad \text{for every physically realizable } P_t$$

$$\Longrightarrow$$

$$A_t^U(P_t) \neq \emptyset.$$

The existence of an unblocked strategically relevant outlier marks the failure of robust positive-development blocking. Its systemic consequence depends on whether unblocking produces a decisively weighted loyalty reversal and ultimately drives

$$S_{t+1} < S_c.$$

An unblocked outlier may remain system-loyal because

$$U_{i,t}^{sys}(P_t) \geq \widetilde{O}_i^C(P_t)$$

may continue to hold.

An individual outlier's loyalty reversal becomes systemically reproduction-destroying when the resulting conduct is sufficient to drive the realized configuration below the reproduction threshold.

Accordingly,

$$\boxed{\bar{P}_t < P_t^B}$$

marks the physical opening of a strategic breach channel. The collapse branch is reached when the realized systemic configuration satisfies

$$S_{t+1} < S_c.$$

The strategic sequence is:

$$\begin{gathered}\text{loss of blocking capacity}\\ \Downarrow \\ \text{strategically relevant outlier unblocking}\\ \Downarrow \\ \text{possible loyalty reversal and strategic breach}\\ \Downarrow \\ \text{possible reproduction failure if } S_{t+1} < S_c.\end{gathered}$$

Whether the opened breach channel becomes reproduction-destroying depends on the realized conduct of the unblocked agents, their systemic weight, and the resulting effect on the reproduction condition.

The next subsection therefore returns to the reproduction threshold itself and defines the minimum developmental effort and physically feasible crisis configurations under which the party-state can continue to reproduce despite exhausted development.

4.6 Minimum Reproduction Effort and Crisis Containment

Systemic reproduction may continue within the exhausted-development crisis regime through crisis containment. The exhausted-development condition

$$g_t^A \leq 0$$

means that positive calibrated development is no longer attainable. It does not itself imply reproduction failure. The realized branch is determined by whether the physically feasible crisis configuration satisfies

$$S_{t+1} \geq S_c$$

or violates that threshold.

The relevant question is therefore no longer whether the system can generate positive calibrated growth. The relevant question is whether, at a given physically realizable disciplinary level, sufficient developmental effort remains available to keep systemic stability above the reproduction threshold.

For any physically realizable disciplinary level

$$P \in [0, \bar{P}_t],$$

define the reproduction-feasible effort set as

$$\mathcal{X}_t^R(P) = \left\{X \in [0, \bar{X}_t] \,:\, g_t(X, P) > -1,\; S_{t+1}(X, P) \geq S_c\right\}.$$

The condition

$$g_t(X, P) > -1$$

preserves positive next-period systemic developedness:

$$D_{t+1}(X, P) = D_t \left[1 + g_t(X, P)\right] > 0.$$

The relevant state transitions are

$$g_t(X, P) = \Gamma(X, g_t^{\max}) - E_t(P) - K(P),$$

$$D_{t+1}(X, P) = D_t \left[1 + g_t(X, P)\right],$$

$$\rho_{t+1}(X, P) = \frac{D_{t+1}(X, P)}{D_{t+1}^w},$$

and

$$S_{t+1}(X, P) = H\left(\rho_{t+1}(X, P), P\right).$$

Define the minimum aggregate developmental effort required for reproduction at disciplinary level

$$P$$

as

$$X_t^R(P) = \begin{cases} \min \mathcal{X}_t^R(P), & \text{if } \mathcal{X}_t^R(P) \neq \emptyset, \\ +\infty, & \text{if } \mathcal{X}_t^R(P) = \emptyset. \end{cases}$$

Assumption 4.16 (Attainment of Minimum Reproduction Effort)**.** Whenever

$$\mathcal{X}_t^R(P) \neq \emptyset,$$

its minimum exists and is attained.

For fixed

$$P,$$

the reproduction-feasible effort set has an upper-set structure.

If

$$X' \geq X,$$

then, because

$$\Gamma_X > 0,$$

$$g_t(X', P) \geq g_t(X, P).$$

Hence,

$$g_t(X, P) > -1$$

implies

$$g_t(X', P) > -1.$$

Moreover,

$$D_{t+1}(X', P) \geq D_{t+1}(X, P),$$

and therefore

$$\rho_{t+1}(X', P) \geq \rho_{t+1}(X, P).$$

Since

$$H_\rho > 0,$$

it follows that

$$S_{t+1}(X', P) \geq S_{t+1}(X, P).$$

Thus,

$$X \in \mathcal{X}_t^R(P) \quad \Longrightarrow \quad [X, \bar{X}_t] \subseteq \mathcal{X}_t^R(P).$$

Under Assumption 4.16, whenever

$$X_t^R(P) < +\infty,$$

the reproduction-feasible effort set is therefore

$$\mathcal{X}_t^R(P) = \left[X_t^R(P), \bar{X}_t\right].$$

Consequently, for fixed physically realizable

$$P,$$

and any

$$X \in [0, \bar{X}_t]$$

satisfying

$$g_t(X, P) > -1,$$

the reproduction condition is characterized by

$$S_{t+1}(X, P) \geq S_c \quad \Longleftrightarrow \quad X \geq X_t^R(P),$$

whenever

$$X_t^R(P) < +\infty.$$

If

$$X_t^R(P) = +\infty,$$

then no physically accessible aggregate developmental effort can preserve systemic reproduction at disciplinary level

$$P.$$

The minimum-reproduction-effort construction is therefore conditional on realized discipline. A deterioration in physically realizable discipline may raise the amount of developmental effort required to preserve reproduction, while sufficiently strong realized discipline may reduce that required effort.

The relationship between discipline and minimum reproduction effort is generally non-monotonic because

$$P$$

affects systemic stability directly while also affecting calibrated development through

$$E_t(P) + K(P).$$

The relevant object is therefore the threshold

$$X_t^R(P)$$

generated by the complete reproduction mapping.

Definition 4.17 (Physical Crisis-Feasible Set)**.** The *physical crisis-feasible set* is

$$\mathscr{F}_t^C = \big\{(X, P) \,:\, 0 \leq X \leq \bar{X}_t,\ 0 \leq P \leq \bar{P}_t,\ g_t(X, P) > -1\big\}.$$

The set

$$\mathscr{F}_t^C$$

contains all effort-discipline configurations physically available to the party-state that preserve positive next-period systemic developedness.

The superscript

$$C$$

denotes the crisis domain only.

A realized crisis configuration is written

$$(X_t^C, P_t^C) \in \mathcal{F}_t^C.$$

The notation

$$P_t^C$$

denotes the disciplinary level actually realized in the crisis configuration.

Likewise,

$$X_t^C$$

denotes realized crisis-period aggregate developmental effort.

Under the exhausted-development regime,

$$g_t^A \leq 0,$$

Proposition 4.3 implies that every physically feasible crisis configuration satisfies

$$g_t(X_t^C, P_t^C) \leq 0.$$

Thus,

$$-1 < g_t(X_t^C, P_t^C) \leq 0$$

for every

$$(X_t^C, P_t^C) \in \mathcal{F}_t^C$$

under exhausted development.

The party-state may therefore remain physically reproduced even though systemic developedness is stationary or declining.

Definition 4.18 (Crisis-Containment Configuration)**.** Under the exhausted-development regime,

$$g_t^A \leq 0,$$

a physically feasible realized configuration

$$(X_t^C, P_t^C) \in \mathcal{F}_t^C$$

is a *crisis-containment configuration* when

$$S_{t+1}(X_t^C, P_t^C) \geq S_c.$$

Crisis containment is a reproduction-preserving configuration under exhausted development in which calibrated growth may be non-positive while the party-state remains above the systemic reproduction threshold:

$$S_{t+1}(X_t^C, P_t^C) \geq S_c.$$

Proposition 4.19 (Characterization of Crisis Containment). *Under the exhausted-development regime,*

$$g_t^A \leq 0,$$

let

$$(X_t^C, P_t^C) \in \mathcal{F}_t^C$$

be a physically feasible realized crisis configuration.

Then

$$(X_t^C, P_t^C)$$

is a crisis-containment configuration if and only if

$$X_t^C \geq X_t^R(P_t^C).$$

If, in addition,

$$P_t^C \geq P_t^B,$$

then all strategically relevant outliers are fully blocked.

If instead

$$P_t^C < P_t^B,$$

full strategic blocking fails, but the configuration remains a crisis-containment configuration whenever

$$S_{t+1}(X_t^C, P_t^C) \geq S_c.$$

Proof. Because

$$(X_t^C, P_t^C) \in \mathcal{F}_t^C,$$

we have

$$0 \leq X_t^C \leq \bar{X}_t, \qquad 0 \leq P_t^C \leq \bar{P}_t,$$

and

$$g_t(X_t^C, P_t^C) > -1.$$

For fixed

$$P_t^C,$$

the attained minimum-reproduction-effort construction gives

$$S_{t+1}(X_t^C, P_t^C) \geq S_c$$

if and only if

$$X_t^C \geq X_t^R(P_t^C),$$

whenever

$$X_t^R(P_t^C) < +\infty.$$

If

$$X_t^R(P_t^C) = +\infty,$$

no

$$X_t^C \in [0, \bar{X}_t]$$

can satisfy the reproduction condition, so no crisis-containment configuration exists at that disciplinary level.

Hence

$$(X_t^C, P_t^C)$$

is a crisis-containment configuration if and only if

$$X_t^C \geq X_t^R(P_t^C).$$

If additionally

$$P_t^C \geq P_t^B,$$

then by Definition 3.56, all strategically relevant outliers satisfy

$$\widetilde{O}_i^C(P_t^C) \leq 0.$$

If instead

$$P_t^C < P_t^B,$$

Lemma 4.14 implies, when

$$A_t^B \neq \emptyset,$$

that at least one strategically relevant outlier is unblocked.

Strategic-outlier unblocking remains compatible with crisis containment while

$$S_{t+1}(X_t^C, P_t^C) \geq S_c.$$

Under this condition, the realized configuration remains reproduction-preserving. □

The distinction between crisis containment and the positive-development reproductive-growth regime is fundamental.

Under the positive-development regime,

$$g_t^A > 0,$$

the system can realize positive calibrated development while reproducing.

Under crisis containment,

$$g_t^A \leq 0,$$

positive calibrated development is no longer attainable, but the realized configuration still satisfies

$$S_{t+1} \geq S_c.$$

Thus:

$$\boxed{\text{positive-development reproduction} \neq \text{crisis containment.}}$$

During crisis containment, realized discipline may continue to change through the adaptive voice process while disciplinary capacity and the system-optimal target continue to evolve.

The term denotes the reproduction-preserving branch of the exhausted-development dynamic:

$$g_t^A \leq 0, \qquad (X_t^C, P_t^C) \in \mathcal{F}_t^C, \qquad S_{t+1}(X_t^C, P_t^C) \geq S_c.$$

The role of minimum reproduction effort can be seen directly under a maintained maximum-effort crisis configuration.

If system-loyal agents continue to supply

$$X_t^C = \bar{X}_t,$$

then crisis containment at realized discipline

$$P_t^C$$

is possible if and only if

$$\bar{X}_t \geq X_t^R(P_t^C).$$

If instead

$$\bar{X}_t < X_t^R(P_t^C),$$

then even maximum accessible aggregate developmental effort is insufficient to preserve systemic reproduction at that realized disciplinary level.

Likewise, if

$$X_t^R(P_t^C) = +\infty,$$

reproduction is infeasible at that disciplinary level regardless of available developmental effort.

The crisis-containment problem therefore separates three distinct questions:

Is the configuration physically realizable?

This requires

$$(X_t^C, P_t^C) \in \mathcal{F}_t^C.$$

Is developmental effort sufficient for reproduction at realized discipline?

This requires

$$X_t^C \geq X_t^R(P_t^C).$$

Are all strategically relevant outliers fully blocked?

This requires

$$P_t^C \geq P_t^B.$$

The third condition is not necessary for the mere continuation of current reproduction.

If

$$P_t^C < P_t^B,$$

a strategic breach channel is open, but crisis containment continues as long as the realized systemic consequence remains

$$S_{t+1}(X_t^C, P_t^C) \geq S_c.$$

The collapse branch begins only when the realized configuration fails this reproduction condition.

4.7 Collapse Dynamic and Endogenous Crisis Deepening

The preceding results distinguish the exhaustion of positive development from the actual failure of systemic reproduction.

Under exhausted development,

$$g_t^A \leq 0,$$

the party-state may remain in crisis containment as long as the realized physically feasible configuration satisfies

$$S_{t+1}(X_t^C, P_t^C) \geq S_c.$$

Collapse begins only when this reproduction condition fails:

$$S_{t+1}(X_t^C, P_t^C) < S_c.$$

The transition toward collapse may nevertheless become endogenously self-deepening once disciplinary capacity becomes binding below the system-optimal disciplinary level.

Recall that

$$P_t^\star$$

is the unconstrained system-optimal disciplinary level and

$$P_t^A = \min\{P_t^\star, \bar{P}_t\}$$

is the attainable optimal disciplinary level.

The disciplinary-capacity shortfall is

$$\Delta_t^P = P_t^\star - P_t^A = \left(P_t^\star - \bar{P}_t\right)_+.$$

When

$$\Delta_t^P > 0,$$

disciplinary capacity is binding at the developmental optimum:

$$P_t^A = \bar{P}_t < P_t^\star.$$

This creates a developmental loss even before any strategic-outlier breach becomes reproduction-destroying.

Definition 4.20 (Capacity-Shortfall Developmental Loss)**.** The *capacity-shortfall developmental loss* is

$$\Lambda_t^P = G_t(P_t^\star) - G_t(P_t^A).$$

Proposition 4.21 (Developmental Loss from Binding Disciplinary Capacity)**.** *Under the maintained strict convexity of*

$$B_t(P) = E_t(P) + K(P)$$

and uniqueness of

$$P_t^\star,$$

the capacity-shortfall developmental loss satisfies

$$\Lambda_t^P \geq 0.$$

Moreover,

$$\Delta_t^P = 0 \quad \Longrightarrow \quad \Lambda_t^P = 0,$$

whereas

$$\Delta_t^P > 0 \quad \Longrightarrow \quad \Lambda_t^P > 0.$$

Hence, when disciplinary capacity is binding below the system-optimal level,

$$G_t(P_t^A) < G_t(P_t^\star).$$

Proof. By definition,

$$P_t^\star = \arg\max_{P \geq 0} G_t(P).$$

Therefore,

$$G_t(P_t^\star) \geq G_t(P_t^A),$$

which implies

$$\Lambda_t^P \geq 0.$$

If

$$\Delta_t^P = 0,$$

then

$$P_t^A = P_t^\star,$$

and therefore

$$\Lambda_t^P = 0.$$

If instead

$$\Delta_t^P > 0,$$

then

$$P_t^A = \bar{P}_t < P_t^\star.$$

Because

$$B_t(P)$$

is strictly convex and has the unique minimizer

$$P_t^\star,$$

the corresponding developmental function

$$G_t(P) = g_t^{\max} - B_t(P)$$

has a unique maximum at

$$P_t^\star.$$

Hence

$$G_t(P_t^A) < G_t(P_t^\star),$$

so

$$\Lambda_t^P > 0.$$

□

The developmental effect of insufficient discipline can be stated more strongly on the capacity-constrained side of the optimum.

Because

$$P_t^\star$$

is the unique interior maximizer of

$$G_t(P),$$

strict convexity of

$$B_t(P)$$

implies that

$$G_t'(P) > 0$$

for

$$0 \le P < P_t^\star.$$

Therefore, for any

$$0 \le P' < P < P_t^\star,$$

$$G_t(P') < G_t(P).$$

Thus, once realized discipline is constrained to the lower side of

$$P_t^\star,$$

a further decline in realized discipline unambiguously lowers full-effort calibrated development.

This removes the ambiguity that arises from examining

$$E_t(P)$$

and

$$K(P)$$

separately.

A fall in discipline raises residual deviation incentives:

$$P' \le P \quad \Longrightarrow \quad r_i(P', D_t, h_i) \ge r_i(P, D_t, h_i),$$

and therefore weakly raises deviation-related harm:

$$E_t(P') \ge E_t(P).$$

At the same time, lower discipline may reduce direct disciplinary cost

$$K(P).$$

However, on the lower side of the unique system optimum,

$$P < P_t^\star,$$

the complete calibrated effect is already summarized by

$$G_t'(P) > 0.$$

Hence,

$$P' < P < P_t^\star$$

implies

$$G_t(P') < G_t(P)$$

after both deviation harm and disciplinary cost have been jointly accounted for.

The residual-deviation mechanism established in Proposition 4.4 therefore operates as one component of a broader, unambiguous developmental deterioration once insufficient command capacity constrains realized discipline below

$$P_t^\star.$$

The capacity shortfall also generates an intertemporal crisis-deepening effect.

To isolate this effect, compare two contemporaneous disciplinary paths under maximum accessible aggregate developmental effort.

The first is the unconstrained system-optimal benchmark:

$$P_t = P_t^\star.$$

Define the associated next-period benchmark states:

$$D_{t+1}^\star = D_t\left[1 + G_t(P_t^\star)\right],$$

$$\rho_{t+1}^\star = \frac{D_{t+1}^\star}{D_{t+1}^w},$$

$$S_{t+1}^\star = H\left(\rho_{t+1}^\star, P_t^\star\right),$$

and

$$\bar{P}_{t+1}^\star = \bar{P}\left(D_{t+1}^\star, \rho_{t+1}^\star, S_{t+1}^\star\right).$$

These quantities define a counterfactual no-shortfall benchmark. They do not imply that

$$P_t^\star$$

is physically realizable when

$$\Delta_t^P > 0.$$

The second path uses the attainable optimal disciplinary level:

$$P_t = P_t^A.$$

Whenever

$$G_t(P_t^A) > -1,$$

define:

$$D_{t+1}^A = D_t\left[1 + G_t(P_t^A)\right],$$

$$\rho_{t+1}^A = \frac{D_{t+1}^A}{D_{t+1}^w},$$

$$S_{t+1}^A = H\left(\rho_{t+1}^A, P_t^A\right),$$

and

$$\bar{P}_{t+1}^A = \bar{P}\left(D_{t+1}^A, \rho_{t+1}^A, S_{t+1}^A\right).$$

Proposition 4.22 (Endogenous Capacity-Shortfall Feedback)**.** *Suppose*

$$\Delta_t^P > 0$$

and

$$G_t(P_t^A) > -1.$$

Then:

$$P_t^A < P_t^\star,$$

$$G_t(P_t^A) < G_t(P_t^\star),$$

$$D_{t+1}^A < D_{t+1}^\star,$$

$$\rho_{t+1}^A < \rho_{t+1}^\star,$$

$$S_{t+1}^A < S_{t+1}^\star,$$

and

$$\bar{P}_{t+1}^A < \bar{P}_{t+1}^\star.$$

Hence a binding disciplinary-capacity shortfall lowers next-period disciplinary capacity relative to the contemporaneous counterfactual path on which the system-optimal disciplinary level remains realizable.

Proof. If

$$\Delta_t^P > 0,$$

then

$$P_t^A = \bar{P}_t < P_t^\star.$$

By Proposition 4.21,

$$G_t(P_t^A) < G_t(P_t^\star).$$

Therefore,

$$D_{t+1}^A = D_t\left[1 + G_t(P_t^A)\right] < D_t\left[1 + G_t(P_t^\star)\right] = D_{t+1}^\star.$$

Since both paths face the same advanced world developedness frontier

$$D_{t+1}^w,$$

it follows that

$$\rho_{t+1}^A < \rho_{t+1}^\star.$$

Moreover,

$$\rho_{t+1}^A < \rho_{t+1}^\star$$

and

$$P_t^A < P_t^\star.$$

Since

$$H_\rho > 0$$

and

$$H_P > 0,$$

we obtain

$$S_{t+1}^A = H\left(\rho_{t+1}^A, P_t^A\right) < H\left(\rho_{t+1}^\star, P_t^\star\right) = S_{t+1}^\star.$$

Finally, disciplinary capacity is strictly increasing in all three state arguments:

$$\bar{P}_D > 0, \qquad \bar{P}_\rho > 0, \qquad \bar{P}_S > 0.$$

Because

$$D^A_{t+1} < D^\star_{t+1},$$

$$\rho^A_{t+1} < \rho^\star_{t+1},$$

and

$$S^A_{t+1} < S^\star_{t+1},$$

it follows that

$$\bar{P}^A_{t+1} < \bar{P}^\star_{t+1}.$$

□

Proposition 4.22 establishes a contemporaneous counterfactual causal effect by comparing the constrained path with the no-shortfall benchmark.

Once command capacity becomes binding below

$$P^\star_t,$$

the capacity constraint itself places the party-state on a path with lower calibrated development, lower next-period developedness, lower relative developedness, lower stability, and lower subsequent disciplinary capacity than would obtain under contemporaneous realization of the system-optimal disciplinary level.

Thus insufficient command capacity can reproduce part of the conditions that make future command capacity weaker.

The endogenous feedback is:

$$\Delta^P_t > 0$$

$$\Downarrow$$

$$P^A_t = \bar{P}_t < P^\star_t$$

$$\Downarrow$$

$$G_t(P^A_t) < G_t(P^\star_t)$$

$$\Downarrow$$

$$D^A_{t+1} < D^\star_{t+1}$$

$$\Downarrow$$

$$\rho^A_{t+1} < \rho^\star_{t+1}$$

$$\Downarrow$$

$$S^A_{t+1} < S^\star_{t+1}$$

$$\Downarrow$$

$$\bar{P}^A_{t+1} < \bar{P}^\star_{t+1}.$$

This feedback operates in addition to the persistent relative developmental erosion already generated by

$$g_t^A \leq 0$$

against an advancing world frontier.

Combined with the net dynamic command-capacity erosion condition of Assumption 4.7, the capacity-shortfall feedback may therefore deepen the erosion of the party-state's inherited command-capacity reserve.

The party-state enters the collapse branch only when the realized crisis configuration actually fails the reproduction threshold.

Proposition 4.23 (Collapse Dynamic under Exhausted Development)**.** *Under the exhausted-development regime,*

$$g_t^A \leq 0,$$

let

$$(X_t^C, P_t^C) \in \mathscr{F}_t^C$$

be a physically feasible realized crisis configuration.

The party-state enters a collapse dynamic if and only if

$$S_{t+1}(X_t^C, P_t^C) < S_c.$$

Equivalently,

$$X_t^C < X_t^R(P_t^C).$$

This may occur even under maximum accessible crisis effort,

$$X_t^C = \bar{X}_t,$$

whenever

$$\bar{X}_t < X_t^R(P_t^C).$$

A disciplinary-capacity shortfall,

$$\Delta_t^P > 0,$$

or a strategic-blocking failure,

$$P_t^C < P_t^B,$$

may contribute to the dynamic that produces this reproduction failure, but neither condition is sufficient for collapse by itself.

Proof. By Definition 4.18, a physically feasible realized crisis configuration is reproduction-preserving if and only if

$$S_{t+1}(X_t^C, P_t^C) \geq S_c.$$

Therefore the complementary reproduction-failure condition is

$$S_{t+1}(X_t^C, P_t^C) < S_c.$$

By the attained minimum-reproduction-effort characterization developed above, for fixed physically realizable

$$P_t^C,$$

the reproduction condition satisfies

$$S_{t+1}(X_t^C, P_t^C) \geq S_c \quad \Longleftrightarrow \quad X_t^C \geq X_t^R(P_t^C),$$

with

$$X_t^R(P_t^C) = +\infty$$

representing the case in which no physically accessible developmental effort can preserve reproduction at that disciplinary level.

Hence,

$$S_{t+1}(X_t^C, P_t^C) < S_c \quad \Longleftrightarrow \quad X_t^C < X_t^R(P_t^C).$$

If

$$X_t^C = \bar{X}_t$$

but

$$\bar{X}_t < X_t^R(P_t^C),$$

then even maximum accessible aggregate developmental effort is insufficient to preserve reproduction.

A positive disciplinary-capacity shortfall,

$$\Delta_t^P > 0,$$

creates an unavoidable departure from the system-optimal disciplinary level and generates the developmental and capacity feedback established in Proposition 4.22.

Likewise,

$$P_t^C < P_t^B$$

produces strategic-outlier unblocking when

$$A_t^B \neq \varnothing.$$

These mechanisms deepen the crisis, and the party-state enters the collapse dynamic when their realized systemic consequences produce

$$S_{t+1}(X_t^C, P_t^C) < S_c.$$

□

The crisis mechanism can therefore deepen through several mutually reinforcing channels without identifying any intermediate channel with collapse itself.

The complete sequence is:

exhausted calibrated development

$$\Downarrow$$

persistent relative developmental erosion

$$\Downarrow$$

under the net erosion condition,

erosion of dynamic command-capacity reserve

$$\Downarrow$$

possible binding disciplinary capacity

$$\Downarrow$$

$$\Delta_t^P > 0 \quad \Rightarrow \quad P_t^A < P_t^\star$$

$$\Downarrow$$

capacity-shortfall developmental loss

$$\Downarrow$$

lower developedness, relative developedness, stability, and future capacity

$$\Downarrow$$

possible strategic-outlier unblocking and rising residual deviation pressure

$$\Downarrow$$

$$\begin{cases} S_{t+1} \geq S_c, & \text{crisis containment,} \\ S_{t+1} < S_c, & \text{collapse dynamic.} \end{cases}$$

The central point is that the nomenklatura may continue to possess an interest in party-state reproduction and may continue to exert adaptive pressure toward stronger disciplinary calibration even while the party-state loses the material and organizational capacity required to realize that calibration.

The collapse branch arises when the realized crisis configuration violates the systemic reproduction threshold, while reproduction-preserving nomenklatura interests may persist.

The crisis process shows how developmental exhaustion, relative erosion, command-capacity erosion, constrained discipline, deviation pressure, and possible strategic breach may progressively undermine the party-state's ability to reproduce itself.

4.8 Crisis Bifurcation

The exhausted-development crisis regime can now be summarized as a dynamic bifurcation between continued systemic reproduction and reproduction failure.

The common developmental condition for both branches is

$$g_t^A \leq 0.$$

This condition means that positive calibrated development is no longer physically attainable. It does not determine whether the party-state immediately collapses.

For any physically feasible realized crisis configuration

$$(X_t^C, P_t^C) \in \mathscr{F}_t^C,$$

the reproduction threshold determines the realized branch:

$$S_{t+1}(X_t^C, P_t^C) \gtreqless S_c.$$

The two branches are therefore:

$$\boxed{\begin{cases} \text{Crisis containment,} & g_t^A \le 0, \\ & (X_t^C, P_t^C) \in \mathscr{F}_t^C, \\ & S_{t+1}(X_t^C, P_t^C) \ge S_c; \\ \text{Collapse dynamic,} & g_t^A \le 0, \\ & (X_t^C, P_t^C) \in \mathscr{F}_t^C, \\ & S_{t+1}(X_t^C, P_t^C) < S_c. \end{cases}}$$

By the minimum-reproduction-effort construction, the same bifurcation can be written:

$$\boxed{\begin{cases} X_t^C \ge X_t^R(P_t^C), & \text{crisis containment,} \\ X_t^C < X_t^R(P_t^C), & \text{collapse dynamic.} \end{cases}}$$

The two branches are exhaustive for every physically feasible realized crisis configuration.

They are also mutually exclusive.

The crisis-deepening mechanisms developed above affect which branch becomes realized, but they must not be confused with the branch criterion itself.

A positive disciplinary-capacity shortfall,

$$\Delta_t^P > 0,$$

means that the unconstrained system-optimal disciplinary level

$$P_t^\star$$

is physically unattainable.

It generates an unavoidable capacity-induced tracking error and a positive developmental loss:

$$\Lambda_t^P > 0.$$

Through Proposition 4.22, this may place the party-state on a path with lower next-period developedness, relative developedness, stability, and disciplinary capacity relative to the contemporaneous no-shortfall benchmark.

When

$$\Delta_t^P > 0,$$

the realized branch remains determined by the reproduction condition

$$S_{t+1}(X_t^C, P_t^C) \gtreqless S_c.$$

Likewise,

$$P_t^C < P_t^B$$

means that full strategic-outlier blocking fails at the realized disciplinary level.

If

$$\bar{P}_t < P_t^B,$$

full strategic blocking is physically impossible at every realizable disciplinary level.

These conditions open or widen a strategic breach channel, while the collapse branch is determined by

$$S_{t+1}(X_t^C, P_t^C) < S_c.$$

The exhausted-development crisis can therefore contain substantial internal deterioration before actual collapse.

The party-state may simultaneously experience:

$$\rho_{t+1} < \rho_t,$$

$$M_{t+1}^P < M_t^P$$

under the net command-capacity erosion condition,

$$\Delta_t^P > 0,$$

and

$$A_t^U(P_t^C) \neq \varnothing,$$

while still remaining temporarily in the containment branch if

$$S_{t+1}(X_t^C, P_t^C) \geq S_c.$$

This is precisely why crisis containment must be distinguished from positive-development reproduction.

Containment means that systemic reproduction continues despite the exhaustion of positive calibrated development and despite possible deterioration in the developmental and disciplinary conditions supporting reproduction.

Collapse occurs only when those accumulated pressures become sufficient to push the realized configuration below the reproduction threshold.

Theorem 4.24 (Crisis Bifurcation). *Suppose the party-state is in the exhausted-development regime:*

$$g_t^A \leq 0.$$

Let

$$(X_t^C, P_t^C) \in \mathscr{F}_t^C$$

be any physically feasible realized crisis configuration.

Then exactly one of the following two branches obtains.

Branch I: Crisis Containment.

If

$$S_{t+1}(X_t^C, P_t^C) \geq S_c,$$

equivalently,

$$X_t^C \geq X_t^R(P_t^C),$$

then

$$(X_t^C, P_t^C)$$

is a crisis-containment configuration and the party-state continues to reproduce.

Branch II: Collapse Dynamic.

If

$$S_{t+1}(X_t^C, P_t^C) < S_c,$$

equivalently,

$$X_t^C < X_t^R(P_t^C),$$

then the party-state enters the collapse dynamic.

In particular, under a maintained maximum-effort crisis configuration,

$$X_t^C = \bar{X}_t,$$

the collapse branch obtains whenever

$$\bar{X}_t < X_t^R(P_t^C).$$

A positive disciplinary-capacity shortfall,

$$\Delta_t^P > 0,$$

a realized strategic-blocking failure,

$$P_t^C < P_t^B,$$

or a physical strategic-blocking-capacity failure,

$$\bar{P}_t < P_t^B,$$

may contribute to the mechanisms that move the realized configuration from Branch I to Branch II, but none of these conditions is sufficient for Branch II independently of the reproduction-threshold condition.

Proof. Let

$$g_t^A \leq 0$$

and let

$$(X_t^C, P_t^C) \in \mathscr{F}_t^C$$

be a physically feasible realized crisis configuration.

Because

$$S_c$$

is the critical reproduction threshold, exactly one of the following two mutually exclusive conditions holds:

$$S_{t+1}(X_t^C, P_t^C) \geq S_c,$$

or

$$S_{t+1}(X_t^C, P_t^C) < S_c.$$

Suppose first that

$$S_{t+1}(X_t^C, P_t^C) \geq S_c.$$

By Definition 4.18, the realized configuration is a crisis-containment configuration.

By Proposition 4.19, this is equivalent to

$$X_t^C \geq X_t^R(P_t^C).$$

Hence the party-state remains above the reproduction threshold and continues to reproduce under exhausted-development conditions.

This is Branch I.

Suppose instead that

$$S_{t+1}(X_t^C, P_t^C) < S_c.$$

By Proposition 4.23, the party-state enters the collapse dynamic.

The same proposition and the minimum-reproduction-effort construction imply equivalently:

$$X_t^C < X_t^R(P_t^C).$$

This is Branch II.

If the maintained crisis configuration satisfies

$$X_t^C = \bar{X}_t$$

and

$$\bar{X}_t < X_t^R(P_t^C),$$

then

$$X_t^C < X_t^R(P_t^C),$$

so Branch II follows even though maximum accessible aggregate developmental effort is being supplied.

Now consider the intermediate crisis-deepening conditions.

If

$$\Delta_t^P > 0,$$

then the system-optimal disciplinary level is physically unattainable and, by Proposition 4.21, the party-state incurs a positive capacity-shortfall developmental loss.

By Proposition 4.22, this capacity constraint may deepen the crisis relative to the no-shortfall counterfactual.

However, if the realized configuration nevertheless satisfies

$$S_{t+1}(X_t^C, P_t^C) \geq S_c,$$

the system remains in Branch I.

Similarly, if

$$P_t^C < P_t^B,$$

then, when

$$A_t^B \neq \emptyset,$$

Lemma 4.14 implies that at least one strategically relevant outlier is unblocked.

If the stronger capacity condition

$$\bar{P}_t < P_t^B$$

holds, Corollary 4.15 implies that full strategic blocking is physically impossible.

Neither condition by itself determines Branch II.

The system enters Branch II only if the realized systemic consequences satisfy

$$S_{t+1}(X_t^C, P_t^C) < S_c.$$

Thus the two branches are exhaustive and mutually exclusive, and the reproduction threshold provides the final branch criterion. □

The crisis bifurcation can therefore be represented as:

$$g_t^A \leq 0$$

$$\Downarrow$$

persistent relative developmental erosion

$$\Downarrow$$

possible cumulative erosion of command-capacity reserve

$$\Downarrow$$

possible capacity shortfall, constrained discipline, and strategic unblocking

$$\Downarrow$$

$$\begin{cases} S_{t+1} \geq S_c, & \text{crisis containment,} \\ S_{t+1} < S_c, & \text{collapse dynamic.} \end{cases}$$

The nomenklatura may retain a reproduction-preserving systemic interest while the developmental and command capacities of the party-state progressively deteriorate.

The final result separates these two objects directly: an interest in systemic reproduction and the material-organizational capacity required to secure systemic reproduction are not the same thing.

Corollary 4.25 (Separation of Reproduction Interest and Command Capacity)**.** *Under exhausted development,*

$$g_t^A \leq 0,$$

the persistence of a reproduction-preserving systemic interest among the relevant nomenklatura body does not imply that the party-state retains the material and organizational command capacity required to secure systemic reproduction.

In particular, reproduction-preserving systemic interest is compatible with any of the following:

$$M_t^P < 0,$$

so that the full effective disciplinary fluctuation neighborhood is no longer physically realizable;

$$\Delta_t^P > 0,$$

so that the unconstrained system-optimal disciplinary level

$$P_t^\star$$

is physically unattainable;

$$\bar{P}_t < P_t^B,$$

so that full strategic-outlier blocking is physically impossible;

and, at a realized crisis disciplinary level

$$P_t^C,$$

$$\bar{X}_t < X_t^R(P_t^C),$$

so that even maximum accessible aggregate developmental effort is insufficient to preserve systemic reproduction.

Hence:

$$\boxed{\textit{reproduction-preserving systemic interest} \;\not\Rightarrow\; \textit{command capacity sufficient for systemic reproduction.}}$$

Proof. Suppose the relevant reproduction-preserving body of the nomenklatura continues to possess a systemic interest in the reproduction of the party-state.

This systemic interest coexists with heterogeneous individual preferences, local deviation, residual free riding, and strategically relevant outliers.

The earlier collapse-conversion results distinguish the systemic interest of the weighted nomenklatura body from the party-state's physical reproduction capacity. Proposition 3.7 establishes the asymptotic result, and Corollary 3.8 implies that, for

$$t \geq T^C,$$

$$\bar{O}_t^{C,w} \leq -\frac{L}{2} < 0.$$

Thus a finite historical domain exists in which the weighted nomenklatura body can retain a strictly negative class-wide collapse-contingent market alternative even while command capacity deteriorates.

The weighted nomenklatura body may retain a reproduction-preserving systemic interest throughout the crisis mechanism.

Moreover, reproduction-preserving agents may continue to supply maximum accessible developmental effort:

$$X_t^C = \bar{X}_t.$$

Likewise, individual adaptive voice may continue to generate disciplinary pressure toward the unconstrained system-optimal level

$$P_t^\star.$$

Neither condition guarantees that the party-state can physically realize the configuration required for reproduction.

Case 1: Loss of Full Dynamic Disciplinary Coverage.

If

$$M_t^P < 0,$$

then

$$\bar{P}_t < P_t^\star + \widehat{R}_t^P.$$

Therefore the full effective disciplinary fluctuation neighborhood around

$$P_t^\star$$

is no longer physically realizable.

This failure can occur even while the exact system-optimal disciplinary level remains attainable:

$$\bar{P}_t \geq P_t^\star.$$

Hence the persistence of reproduction-preserving systemic interest does not imply sufficient capacity to preserve the full normal dynamic disciplinary regime.

Case 2: Loss of the System-Optimal Disciplinary Level.

If

$$\Delta_t^P > 0,$$

then

$$\bar{P}_t < P_t^\star,$$

and therefore

$$P_t^A = \bar{P}_t < P_t^\star.$$

No physically realizable disciplinary level can equal the unconstrained system-optimal level.

Even if adaptive aggregate voice generates raw disciplinary pressure satisfying

$$P_t^\Phi(V_t) \geq P_t^\star,$$

realized discipline remains bounded by

$$P_t = \min\left\{P_t^\Phi(V_t), \bar{P}_t\right\} \leq \bar{P}_t < P_t^\star.$$

Thus,

disciplinary preference and adaptive pressure

do not imply

disciplinary realizability.

By Proposition 4.21, the resulting capacity shortfall also generates a strictly positive developmental loss:

$$\Lambda_t^P > 0.$$

By Proposition 4.22, this places the party-state on a path with lower next-period developedness, relative developedness, stability, and disciplinary capacity than under the contemporaneous no-shortfall benchmark.

Hence reproduction-preserving systemic interest can persist while the command capacity required for system-optimal disciplinary calibration has already been lost.

Case 3: Loss of Strategic-Blocking Capacity.

Suppose

$$\bar{P}_t < P_t^B$$

and

$$A_t^B \neq \emptyset.$$

By Corollary 4.15, every physically realizable disciplinary level satisfies

$$P_t < P_t^B,$$

and therefore full strategic-outlier blocking is physically impossible.

This result remains true even if the nomenklatura's raw voice-induced disciplinary pressure satisfies

$$P_t^\Phi(V_t) \geq P_t^B.$$

The binding physical constraint is:

$$P_t \leq \bar{P}_t < P_t^B.$$

Strategically relevant outliers are therefore necessarily unblocked at every physically realizable disciplinary level.

The party-state remains in crisis containment while

$$S_{t+1}(X_t^C, P_t^C) \geq S_c,$$

even after strategic-blocking capacity has failed.

However, if the resulting strategic breach becomes reproduction-destroying and produces

$$S_{t+1}(X_t^C, P_t^C) < S_c,$$

collapse occurs despite the continued existence of a reproduction-preserving systemic interest among the relevant nomenklatura body.

Case 4: Reproduction Infeasibility despite Maximum Accessible Effort.

Suppose system-loyal agents continue to supply maximum accessible aggregate developmental effort:

$$X_t^C = \bar{X}_t.$$

At the realized disciplinary level

$$P_t^C,$$

if

$$\bar{X}_t < X_t^R(P_t^C),$$

then

$$X_t^C < X_t^R(P_t^C).$$

By Proposition 4.23, this implies

$$S_{t+1}(\bar{X}_t, P_t^C) < S_c.$$

Thus even maximum accessible aggregate developmental effort is insufficient to preserve reproduction at the realized disciplinary level.

The stronger case

$$X_t^R(P_t^C) = +\infty$$

means that no aggregate developmental effort in

$$[0, \bar{X}_t]$$

can preserve reproduction at that disciplinary level.

More generally, if

$$\bar{X}_t < X_t^R(P)$$

for every physically realizable disciplinary level

$$P \in [0, \bar{P}_t]$$

satisfying

$$g_t(\bar{X}_t, P) > -1,$$

then no physically feasible maximum-effort crisis configuration can preserve reproduction.

This is a failure of reproduction feasibility despite the continued supply of all accessible developmental effort.

The four cases establish the separation.

The nomenklatura may continue to possess a systemic interest in reproducing the party-state.

It may continue to supply maximum accessible developmental effort.

Its adaptive voice may continue to push toward the disciplinary level that would best calibrate the system.

Nevertheless, the party-state may lack:

$$\text{capacity for the full dynamic disciplinary neighborhood,}$$

$$\text{capacity to realize } P_t^\star,$$

$$\text{capacity to block strategically relevant outliers,}$$

or ultimately

$$\text{capacity to realize any effort-discipline configuration satisfying } S_{t+1} \geq S_c.$$

Therefore:

$$\text{reproduction-preserving systemic interest} \;\not\Rightarrow\; \text{command capacity sufficient for systemic reproduction.}$$

□

The separation result completes the crisis logic of the model.

The positive-development regime is characterized by the joint attainability of positive calibrated development, sufficient disciplinary capacity, robust dynamic tracking, strategic-outlier blocking, and systemic reproduction.

Its characteristic historical expansion may include catching-up development:

$$g_t > g_t^w,$$

which raises relative developedness:

$$\rho_{t+1} > \rho_t,$$

and supports the accumulation of the material and organizational base underlying command capacity.

The positive-development regime may include both catching-up periods and temporary periods of positive absolute growth with relative developmental erosion:

$$0 < g_t < g_t^w.$$

Such periods remain compatible with continued positive development and systemic reproduction while accumulated developmental and command-capacity margins remain sufficient.

The exhausted-development regime begins only when:

$$g_t^A \leq 0.$$

At that point, every physically feasible realized configuration satisfies:

$$g_t \leq 0,$$

while the advanced world developedness frontier continues to satisfy:

$$g_t^w > 0.$$

Consequently:

$$\rho_{t+1} < \rho_t.$$

Persistent relative developmental erosion then generates a continuing developmental pressure on disciplinary capacity.

Under the net dynamic command-capacity erosion condition:

$$M_{t+1}^P < M_t^P.$$

The party-state may initially absorb this deterioration through an inherited positive command-capacity reserve.

As that reserve is consumed, however, the system may sequentially lose:

full dynamic fluctuation coverage,

then possibly:

attainability of the system-optimal disciplinary level,

and possibly:

capacity for full strategic-outlier blocking.

Once:

$$\Delta_t^P > 0,$$

the realized disciplinary process is separated from its unconstrained system-optimal target by an unavoidable capacity-induced tracking error.

The resulting capacity-shortfall developmental loss:

$$\Lambda_t^P > 0$$

can further weaken developedness, relative developedness, stability, and future disciplinary capacity relative to the no-shortfall counterfactual.

The crisis process may therefore become endogenously self-deepening while the nomenklatura's class-wide reproduction interest persists.

The full model can be summarized by the following dynamic structure:

$$\text{non-generalizable class-wide collapse conversion}$$

$$\Downarrow$$

$$\text{systemic interest in party-state reproduction}$$

$$\Downarrow$$

$$\text{developmental alignment and adaptive disciplinary voice}$$

$$\Downarrow$$

during the expansive historical phase,

$$\text{catching-up development and accumulation of command-capacity reserve}$$

$$\Downarrow$$

$$\text{slowdown and possible relative developmental erosion}$$

$$\Downarrow$$

$$g_t^A \leq 0$$

$$\Downarrow$$

$$\text{persistent relative developmental erosion}$$

$$\Downarrow$$

under the net erosion condition,

$$\text{erosion of inherited command-capacity reserve}$$

$$\Downarrow$$

$$\text{possible constrained discipline and strategic unblocking}$$

$$\Downarrow$$

$$\begin{cases} S_{t+1} \geq S_c, & \text{crisis containment,} \\ S_{t+1} < S_c, & \text{collapse.} \end{cases}$$

The model therefore distinguishes the motive to reproduce a system from the historical capacity to reproduce it.

The party-state may cease to reproduce while the nomenklatura retains a reproduction-preserving systemic interest, because the developmental, disciplinary, organizational, and stability conditions required to realize that interest have themselves become unattainable.